EXHIBIT 1

(OSPA USE ONLY)
Proposal #: 2018.09.017
Account #:
GCA #:
Budget Reviewed by: KA
Date: 9/17/18

PROPOSAL CHECKLIST

OFFICE OF SPONSORED PROJECTS ADMINISTRATION
SOUTHERN ILLINOIS UNIVERSITY CARBONDALE

☒ Preproposal
☐ Proposal
☐ Proposal/Budget Revision
(__________)

☒ New
☐ Supplement __________
☐ Continuation/Renewal __________
(Year ____ or ____)

☒ Research
☐ M01B NSF Code
☐ Training/Credit
☐ Training/Non-Credit
☐ Service

☒ Federal
☐ State
☐ Industry
☐ Foundation
☐ Other

PROJECT INFO

TITLE: Study of Low Rank Approximation of Tensorial Data Set via Non-convex Regularization

FUNDING AGENCY: NSF   **Announcement #** PD18-8069

Deadline Date: 09/17/2018   ☒ Received ☐ Postmarked
Project Dates: 05/15/2019 to 05/14/2022
Amount Requested $ 151,098
SIU Match $ 0.00   Non-SIU Match $ 0.00
If match is included, complete Cost Sharing section below.

F&A Cost Rate 47.5%   F&A Cost $ 48659.--
100 % returned F&A to Dept. Math
____ % returned F&A to Dept.
____ % returned F&A to Dept.
____ % returned F&A to Dept.

Does this proposal include a subcontract? ☒ **No** ☐ **Yes:** If yes, proposal must include the subcontractor scope of work, budget and subcontractor's authorized administrative approval.

CENTER AFFILIATION

	Yes	No	Director Signature
Center for Ecology	☐	☒	
Materials Technology Center	☐	☒	
Center for Fisheries, Aquaculture & Aquatic Sciences	☐	☒	
Cooperative Wildlife Research Laboratory	☐	☒	
Neuroscience Research Center	☐	☒	
STEM Education Research Center	☐	☒	

PERSONNEL

PRINCIPAL INVESTIGATOR/PROJECT DIRECTOR (PI/PD):

Lead PI/PD	Department
Mingqing Xiao	Mathematics

Other PI/PD(s)	Department

NON PI/PD PERSONNEL:

Name	Department

Will compensation to any of these personnel be paid pursuant to the University's policy on overload or extra compensation? ☐ Yes ☒ No

EFFORT

SIUC-FUNDED EFFORT: Required Indicate the percentage of your total appointment that will be committed to this project not including salary reported to the sponsoring agency. If all of your time is charged to the project and paid by the sponsor as salary or is budgeted as cost share and no additional time is allocated to this project, indicate zero.

Name	% of Effort of PI's Total Appointment
Mingqing Xiao	0

COST SHARING OTHER THAN CONTRIBUTED EFFORT

Is Cost Sharing Required by Agency: ☒ No ☐ Yes

If No, cost sharing by SIUC is not allowed without a completed **Voluntary Cost Share Waiver Request.**

If Yes, or with an approved Waiver Request, attach the **Cost Sharing Commitment Form** if you are including cost share other than effort or waived indirect.

Requesting match from the OVCR? ☒ No ☐ Yes

If Yes, attach the **Cost Share Request Form.** This form must be submitted to OSPA with your checklist, not to the OVCR.

DOES THIS PROJECT INVOLVE:

Human Subjects? [X] No [ ] Yes #______

Vertebrate Animals? [X] No [ ] Yes #______

Grad Students? [ ] No [X] Yes # 2

Undergrad Students? [ ] No [X] Yes # 2

Travel to, or Research in, a foreign country; a foreign collaborator; or export of any equipment or materials to a foreign location? [X] No [ ] Yes

Human Stem Cell Research [X] No [ ] Yes #______

Have you read the SIUC Policy on Export Control? [ ] No [X] Yes

Are you aware of any Export control issues with this proposal? [X] No [ ] Yes
If yes, please fill out the **Export Control Disclosure Form**

Is this a Federally Funded research project? [ ] No [X] Yes
If yes, a **Financial Conflict of Interest Form** may be required prior to proposal submission. If funded, FCOI training may be required.

Access to funding will be granted once above required approvals are in place.

Are there Chemical or Biological Hazards, Controlled Substances or Dual Use Research of Concern involved in your project? [X] No [ ] Yes If yes, **click here.**

Does this proposal include the use or development of an unmanned aerial system (UAV, drone)? [X] No [ ] Yes

Does this project involve the creation or modification of curriculum, hiring of faculty or other academic change? [X] No [ ] Yes
If yes, approval from the Provost is required below.

PI/PD SIGNATURES

By signing below, you agree that you have read and approved the contents of this proposal checklist.

NOTE TO PI/PD(s): By signing this transmittal form, you are certifying that 1) the information submitted herein is true, complete, and accurate to the best of your knowledge; 2) any false, fictitious, or fraudulent statements or claims may subject you to criminal, civil or administrative penalties; 3) you agree to accept responsibility for the scientific conduct of the project and to provide progress reports as required by the funding agency; and 4) you are in conformance with SIUC's Conflict of Interest Policy and Financial Conflict of Interest on Federal Grants Policy.

I understand and acknowledge the following: a) if this proposal for external funding includes a request for 100% salary, regardless of the period and duration, from Federal, State and/or other sponsored project(s), I certify that if this project is awarded, I will work exclusively on this project during that 100%-funded period; b) if such award is accepted, it would not allow for vacation/personal time/consulting, or for performing other University activities during the 100% funded period. I understand that any Personnel Request Form will reflect this commitment of effort and that failure to maintain adequate substantiating records, or failure to meet my effort commitments may constitute a violation of University and/or funding agency policy, and could result in serious consequences to the University and me.

Mingqing Xiao 09/13/2018

Signature of Principal Investigator Date | Signature of Principal Investigator

Printed Name of Principal Investigator | Printed Name of Principal Investigator

Signature of Non PI/PD Personnel Date | Signature of Non PI/PD Personnel

Printed Name of Non PI/PD Personnel | Printed Name of Non PI/PD Personnel

FISCAL OFFICE SIGNATURE

By signing this checklist: 1) I confirm that the faculty/staff time commitment will be consistent with faculty/staff assigned effort in the department and will be within the maximum 100% time available for University duties should this proposal be funded; and 2) as a fiscal officer for Southern Illinois University, I agree to the following: (see Fiscal Officer Practices Memo.) a) I am responsible for maintaining the financial resources to meet all commitments and ensuring the propriety of all transactions requested to be posted to the account; b) I am responsible for providing all required information that is necessary for the appropriate maintenance of the University financial systems; c) I am required to follow all applicable rules, regulations, laws related to financial transactions and violation of these rules, regulations, laws may result in revocation of fiscal officership; d) I understand that entering into agreements that are beyond the scope of my authority as outlined by board of Trustees policy may result in personal liability; and e) I may submit and validate my authorization of transactions for the University financial systems through electron means. I am responsible for all fiscal transactions authorized by my electronic signature. It is my responsibility to protect the confidentiality of my unique log-on account and password for these systems.

Mingqing Xiao

Signature of Project Fiscal Office Date | Printed Name of Project Fiscal Officer

OTHER REQUIRED SIGNATURES

As a Unit Officer for Southern Illinois University, I agree to the following: 1)If a Fiscal Officer under my unit responsibility terminates employment with the University, I will reassign the fiscal officer responsibility for each of their accounts to another active University employee; 2) If the accounts are not reassigned within 30 days of the resignation/separation date of the Fiscal Officer, they will be automatically reassigned to the Unit Officer; 3) I have thoroughly reviewed this checklist and accompanying documents and approve the proposed scope of work, level of faculty effort and budget.

9/13/2018

Signature of Chair/Unit Officer Date | Signature of Chair/Unit Officer Date

Printed Name of Chair/Unit Officer | Printed Name of Chair/Unit Officer

Scott E Ishman 9/13/2018

Signature of Dean/Next Level of Authority Date | Signature of Dean/Next Level of Authority Date

Scott E Ishman

Printed Name of Dean/Next Level of Authority | Printed Name of Dean/Next Level of Authority

Sonja Schwartz 9/17/18

Other Required Approvals Date | Director, Office of Sponsored Projects Administration Date

OSPA Use Only

Proposal Submission Date______ Copies Sent: **Submission Method:** [ ] 1st Class [ ] Overnight [X] FastLane [ ] Grants.gov [ ] Pl

Proposals/Supplements/File Updates/Withdrawals | MAIN

Organization: Southern Illinois University at Carbondale

Documents in Progress | Withdrawals In Progress | Submitted Documents

PROPOSAL SUBMISSION CONFIRMATION

Proposal 7904242 has been successfully submitted to NSF

Sep 17 2018 11:19AM EDT

This proposal has now been assigned the following NSF Proposal Number:
1854638

Please make a note of this number, it is the official NSF proposal number.
Your Signature has been recorded

OK

Download Adobe Acrobat Reader for viewing PDF files

SIU Southern Illinois University CARBONDALE OFFICE OF SPONSORED PROJECTS ADMINISTRATION

OSPA ONLY
DRAFT
FINAL
Reviewed By: ____________

AGENCY: NSF, Division of Mathematical Sciences

PROJECT TITLE: Study of low rank approximation of tensorial data set via non-convex regularization

Budget Period: (REQUIRED) 5/15/2019 **through** 5/14/2020

	Category	Fringe Rate	Salary	# Mos	% of Effort	Person/ Months	Requested Salary	Requested Fringe	Total Request
A. Personnel									
Mingqing Xiao	Faculty	58.30%	10,632	1	100%	1	10,632	6,198	16,830
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
Subtotal						1	10,632	6,198	**16,830**
B. Student Personnel									
SEE BUDGET RATES FOR CURRENT GRADUATE STUDENT STIPEND RATES AND PRIMARY CARE FEE									
Graduate Student PhD 1 May-June	Graduate Student		3,488		25%				
Graduate Student PhD 1 July-May	Graduate Student		3,593	2	25%	0.5	1,796	124	1,920
Graduate Student PhD 2	Graduate Student		3,488		25%				
Graduate Student PhD 2	Graduate Student		3,593	2	25%	0.5	1,796	124	1,920
Undergraduate student (10/hr ~8wks)	SELECT								
Undergraduate student (10/hr ~8wks)	SELECT								
Subtotal						1	3,593	248	**3,841**
Subtotal Personnel and Benefits							14,224	6,446	**20,671**

C. Equipment	Qty.	Price Per Item	Total Request
Subtotal			

D.Travel	# of Travelers	Mileage	Lodging	Airfare	Per Diem	Total Request
Conferences	1					1,800
Subtotal						**1,800**

E. Commodities	Qty.	Price Per Item	Total Request
Materials/supplies			200
Subtotal			**200**

F. Other Direct Costs	Total Request
Publication	800
Subtotal	**800**

G. Subawards	Total Request
Subtotal	

	Category	Rate	Total
H. Direct Costs			**23,471**
I. Indirect Costs	Research (On-Campus)	0.475	11,149
J. Total Year One Costs			**34,620**

SIU Southern Illinois University
CARBONDALE OFFICE OF SPONSORED PROJECTS ADMINISTRATION

AGENCY: NSF, Division of Mathematical Sciences

PROJECT TITLE: Study of low rank approximation of tensorial data set via non-convex regularization

Budget Period: (REQUIRED) 5/15/2020 through 5/14/2021

	Category	Fringe Rate	Salary	# Mos	% of Effort	Person/ Months	Requested Salary	Requested Fringe	Total Request
A. Personnel									
Mingqing Xiao	Faculty	58.30%	10,951	1	100%	1	10,951	6,384	17,335
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
Subtotal						1	10,951	6,384	**17,335**
B. Student Personnel								Enter GA Primary Care Fee (see budget rates)	
SEE BUDGET RATES FOR CURRENT GRADUATE STUDENT STIPEND RATES									
Graduate Student PhD 1 May-June	Graduate Student		3,593	1.5	25%	0.375	1,347	74	1,421
Graduate Student PhD 1 July-May	Graduate Student		3,700	7.5	25%	1.875	6,938	256	7,194
Graduate Student PhD 2	Undergraduate Student		3,593	1.5	25%	0.375	1,347	74	1,421
Graduate Student PhD 2	Graduate Student		3,700	7.5	25%	1.875	6,938	256	7,194
Undergraduate student (10/hr ~8wks)	Undergraduate Student		400	8	25%	2	800		800
Undergraduate student (10/hr ~8wks)	Undergraduate Student		400	8	25%	2	800		800
Subtotal						8.5	18,171	660	**18,831**
Subtotal Personnel and Benefits							29,122	7,044	**36,166**

C. Equipment	*Qty.*	*Price Per Item*							Total Request
Subtotal									

D. Travel	*# of Travelers*	*Mileage*	*Lodging*	*Airfare*	*Per Diem*				Total Request
Conferences	1								1,800
Subtotal									**1,800**

E. Commodities	*Qty.*	*Price Per Item*							Total Request
Materials/supplies									200
Subtotal									**200**

F. Other Direct Costs									Total Request
Publication									800
Subtotal									**800**

G. Subawards									Total Request
Subtotal									
H. Direct Costs									**38,966**
	Category	*Rate*							
I. Indirect Costs	Research (On-Campus)	0.475							18,509
J. Total Year Two Costs									**57,475**

SIU Southern Illinois University CARBONDALE
OFFICE OF SPONSORED PROJECTS ADMINISTRATION

AGENCY: NSF, Division of Mathematical Sciences

PROJECT TITLE: Study of low rank approximation of tensorial data set via non-convex regularization

Budget Period: (REQUIRED) 5/15/2021 **through** 5/14/2022

	Category	Fringe Rate	Salary	# Mos	% of Effort	Person/ Months	Requested Salary	Requested Fringe	Total Agency Request
A. Personnel									
Mingqing Xiao	Faculty	58.30%	11,279	1	100%	1	11,279	6,576	17,855
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
	SELECT								
Subtotal						1	11,279	6,576	**17,855**
B. Student Personnel								Enter GA Primary Care Fee (see budget rates)	
SEE BUDGET RATES FOR CURRENT GRADUATE STUDENT STIPEND RATES									
Graduate Student PhD 1 May-June	Graduate Student		3,700	1.5	25%	0.375	1,388	76	1,464
Graduate Student PhD 1 July-May	Graduate Student		3,811	7.5	25%	1.875	7,146	264	7,410
Undergraduate Student	Graduate Student		3,700	1.5	25%	0.375	1,388	76	1,464
Graduate Student PhD 2	Graduate Student		3,811	7.5	25%	1.875	7,146	264	7,410
Undergraduate student (10/hr ~8wks)	Undergraduate Student		400	8	25%	2	800		800
Undergraduate student (10/hr ~8wks)	Undergraduate Student		400	8	25%	2	800		800
Subtotal						8.5	18,668	680	**19,348**
Subtotal Personnel and Benefits							29,947	7,256	**37,203**
C. Equipment	Qty	Price Per Item							Total Agency Request
Subtotal									
D. Travel	# of Travelers	Mileage	Lodging	Airfare	Per Diem				Total Agency Request
Conferences	1								1,800
Subtotal									**1,800**
E. Commodities	Qty	Price Per Item							Total Agency Request
Materials/supplies									200
Subtotal									**200**
F. Other Direct Costs									Total Agency Request
Publication									800
Subtotal									**800**
G. Subawards									Total Agency Request
Subtotal									
H. Direct Costs									**40,003**
	Category	Rate							
I. Indirect Costs	Research (On-Campus)	0.475							19,001
J. Total Year Three Costs									**59,004**

SIU Southern Illinois University
CARBONDALE OFFICE OF SPONSORED PROJECTS ADMINISTRATION

AGENCY: NSF, Division of Mathematical Sciences

PROJECT TITLE:

Budget Period: (REQUIRED) 5/15/19 through 5/14/22

	Person/ Months	Requested Salary	Requested Fringe	Total Request
A. Personnel				
Mingqing Xiao	3	32,861	19,158	52,020
Subtotal	3	32,861	19,158	52,020
B. Student Personnel				
Graduate Student PhD 1 May-June	0.75	2,735	150	2,885
Graduate Student PhD 1 July-May	4.25	15,881	644	16,525
Graduate Student PhD 2	0.75	2,735	150	2,885
Graduate Student PhD 2	4.25	15,881	644	16,525
Undergraduate student (10/hr ~8wks)	4	1,600		1,600
Undergraduate student (10/hr ~8wks)	4	1,600		1,600
Subtotal	18	40,432	1,588	42,020
Subtotal Personnel and Benefits		73,293	20,746	94,039
D. Equipment				Total Request
Subtotal				
E. Travel				Total Request
Conferences				5,400
Subtotal				5,400
F. Commodities				Total Request
Materials/supplies				600
Subtotal				600
G. Other Direct Costs				Total Request
Publication				2,400
Subtotal				2,400
H. Subawards				Total Request
Subtotal				
H. Direct Costs				102,439
I. Indirect Costs				48,659
J. Total Project Costs				151,098

List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:
Not Listed

REVIEWERS NOT TO INCLUDE:
Not Listed

Pursuant to PAPPG Chapter II.C.1.e., each PI, co-PI, and other senior project personnel identified on a proposal must provide collaborator and other affiliations information to help NSF identi

Please complete this template (e.g., Excel, Google Sheets, LibreOffice), save as .xlsx or .xls, and upload directly as a Fastlane Collaborators and Other Affiliations single copy doc.

Do not upload .pdf.

There are five tables:
A: Your Name & Affiliation(s);
B: PhD Advisors/Advisees (all);
C: Collaborators;
D: Co-Editors;
E: Relationships

List names as Last Name, First Name, Middle Initial. Additionally, provide email, organization, and department (optional) to disambiguate common names.

Fixed column widths keep this sheet one page wide; if you cut and paste text, set font size at 10pt or smaller, and abbreviate, where necessary, to make the data fit.

To insert n blank rows, select n row numbers to move down, right click, and choose Insert from the menu.

You may fill-down (crtl-D) to mark a sequence of collaborators, or copy affiliations. Excel has arrows that enable sorting.
"Last active" dates are optional, but will help NSF staff easily determine which information remains relevant for reviewer selection.

Table A: List your Last Name, First Name, Middle Initial, and organizational affiliation (including considered affiliation) in the last 12 months.

A	Your Name:	Your Organizational Affiliation(s), last 12 mo	Last Active Date
	Xiao, Mingqing	Southern Illinois University Carbondale	

Table B: List names as Last Name, First Name, Middle Initial, and provide organizational affiliations, if known, for the following.

G: Your PhD Advisor(s)
T: All your PhD Thesis Advisees
P: Your Graduate Advisors

B	Advisor/Advisee Name:	Organizational Affiliation	*to disambiguate common names* Optional (email, Department)
G:	Basar, Tamer	University of Illinois at Urnana-Champaign	
T:	Basar, Tamer	University of Illinois at Urnana-Champaign	
P:	Basar, Tamer	University of Illinois at Urnana-Champaign	

Table C: List names as Last Name, First Name, Middle Initial, and provide organizational affiliations, if known, for the following.

A: Co-authors on any book, article, report, abstract or paper (with collaboration in last 48 months; publication date may be later).

C: Collaborators on projects, such as funded grants, graduate research or others (in last 48 months).

C	Name:	Organizational Affiliation	*to disambiguate common names* Optional (email, Department)	Last Active
A:	James T. Cronin	Louisiana State University		1/15/16
A:	Xiongping Dai	Nanjing University		6/1/17
A:	Min A	Coppin State University		6/10/13
A:	Yiu-ming Cheung	Hong Kong Baptist University		9/12/13
A:	Xin-zhuang Dong	Qingdao University		5/1/17
A:	Bittany Froese	New Jersey Institute of Technology		2/12/17
A:	Xuejun Gao	Guangdong University of Technology		12/20/14
A:	Yu Huang	Zhongshan University		6/1/17
A:	Tingwen Hang	Texas A&M University at Qatar		6/1/17
A:	Wei Kang	Naval Postgraduate School		4/12/15
A:	Authur J. Krener	Naval Postgraduate School		4/12/15
A:	Buyang Li	Nanjing University		6/1/17
A:	Chuangdong Li	Southwest University		5/10/15
A:	Hailin Liu	Guangdong University of Technology		4/20/14
A:	Adam M. Oberman	McGill University		6/1/17
A:	Wen Li	South China Normal University		2/1/16
A:	John D. Reeve	Southern Illinois University Carbondale		1/15/16
A:	Jens Saak	Max Planck Institute for Dynamics of Complex Technology		7/1/15
A:	Haiwei Sun	University of Macau		6/1/16
A:	Jiechang Wen	Guangdong University of Technology		9/1/14
A:	Dashun Xu	Southern Illinois University Carbondale		1/10/16
A:	Jianhong Xu	Southern Illinois University Carbondale		9/15/14
A:	Liang Xu	Naval Research Laboratory		4/12/15
A:	Bin Zhou	Harbin Institute of Technology		7/1/15
A:	Zhigang Zeng	Huazhong University of Science and Technology		10/1/15
A:	Xiaojun Zhou	Central South University		10/1/15

C:	John D. Reeve	Southern Illinois University Carbondale		8/30/16
C:	Dashun Xu	Southern Illinois University Carbondale		8/30/16
C:	Jianhong Xu	Southern Illinois University Carbondale		7/31/17
A:	Hailing Dong	Shenzhen University		7/31/18

Table D: List editorial board, editor-in-chief and co-editors with whom you interact. An editor-in-chief should list the entire editorial board.

B: Editorial board: Name(s) of editor-in-chief and journal (in past 24 months).

E: Other Co-Editors of journals or collections with whom you directly interacted (in past 24 months).

D	Name:	Organizational Affiliation	*to disambiguate common names* Journal/Collection	Last Active
B:	T. Parisini	Imperial College London	European Journal of Control	
E:	N/A			

Table E: List persons for whom a personal, family, or business relationship would otherwise preclude their service as a reviewer.

R: Additional names for whom some relationship would otherwise preclude their service as a reviewer.

D	Name:	Organizational Affiliation	*to disambiguate common names* Optional (email, Department)	Last Active
R:	N/A			

Not for distribution

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./DUE DATE		☐ Special Exception to Deadline Date Policy	FOR NSF USE ONLY
PD 18-8069	09/17/18		NSF PROPOSAL NUMBER
FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.)			1854638
DMS - CDS&E-MSS			

DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
09/17/2018	2	03040000 DMS	8069	939007555	09/17/2018 11:19am

EMPLOYER IDENTIFICATION NUMBER (EIN) OR TAXPAYER IDENTIFICATION NUMBER (TIN)	SHOW PREVIOUS AWARD NO. IF THIS IS ☐ A RENEWAL ☐ AN ACCOMPLISHMENT-BASED RENEWAL	IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY? YES ☐ NO ☒ IF YES, LIST ACRONYM(S)
376005961		

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE	ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE
Southern Illinois University at Carbondale	Southern Illinois University at Carbondale Ofc. of Sponsored Projects Admin Carbondale, IL. 629014308
AWARDEE ORGANIZATION CODE (IF KNOWN)	
0017582000	
NAME OF PRIMARY PLACE OF PERF	ADDRESS OF PRIMARY PLACE OF PERF, INCLUDING 9 DIGIT ZIP CODE
Southern Illinois University at Carbondale	Southern Illinois University at Carbondale IL ,629014302 ,US.

IS AWARDEE ORGANIZATION (Check All That Apply) ☐ SMALL BUSINESS ☐ FOR-PROFIT ORGANIZATION ☐ MINORITY BUSINESS ☐ WOMAN-OWNED BUSINESS ☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **Study of Low Rank Approximation of Tensorial Data Set via non-convex Regularization**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ 151,099	36 months	05/15/19	

THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW

☐ BEGINNING INVESTIGATOR
☐ DISCLOSURE OF LOBBYING ACTIVITIES
☐ PROPRIETARY & PRIVILEGED INFORMATION
☐ HISTORIC PLACES
☐ VERTEBRATE ANIMALS IACUC App. Date ______
PHS Animal Welfare Assurance Number ______
☒ TYPE OF PROPOSAL **Research**

☐ HUMAN SUBJECTS Human Subjects Assurance Number ______
Exemption Subsection ______ or IRB App. Date ______
☐ INTERNATIONAL ACTIVITIES: COUNTRY/COUNTRIES INVOLVED
☒ COLLABORATIVE STATUS
Not a collaborative proposal

PI/PD DEPARTMENT	PI/PD POSTAL ADDRESS
MATHEMATICS	Department of Mathematics Carbondale, IL 62901 United States
PI/PD FAX NUMBER	
618-453-5300	

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Email Address
PI/PD NAME MingQing Xiao	PhD	1997	618-453-6572	mxiao@math.siu.edu
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

CERTIFICATION PAGE

Certification for Authorized Organizational Representative (or Equivalent) or Individual Applicant

By electronically signing and submitting this proposal, the Authorized Organizational Representative (AOR) or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding conflict of interest (when applicable), drug-free workplace, debarment and suspension, lobbying activities (see below), nondiscrimination, flood hazard insurance (when applicable), responsible conduct of research, organizational support, Federal tax obligations, unpaid Federal tax liability, and criminal convictions as set forth in the NSF Proposal & Award Policies & Procedures Guide (PAPPG). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U.S. Code, Title 18, Section 1001).

Certification Regarding Conflict of Interest

The AOR is required to complete certifications stating that the organization has implemented and is enforcing a written policy on conflicts of interest (COI), consistent with the provisions of PAPPG Chapter IX.A.; that, to the best of his/her knowledge, all financial disclosures required by the conflict of interest policy were made; and that conflicts of interest, if any, were, or prior to the organization's expenditure of any funds under the award, will be, satisfactorily managed, reduced or eliminated in accordance with the organization's conflict of interest policy. Conflicts that cannot be satisfactorily managed, reduced or eliminated and research that proceeds without the imposition of conditions or restrictions when a conflict of interest exists, must be disclosed to NSF via use of the Notifications and Requests Module in FastLane.

Drug Free Work Place Certification

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent), is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Proposal & Award Policies & Procedures Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant is providing the Debarment and Suspension Certification contained in Exhibit II-4 of the Proposal & Award Policies & Procedures Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:

(1) No Federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any Federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.

(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, "Disclosure of Lobbying Activities," in accordance with its instructions.

(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

Certification Regarding Nondiscrimination

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is providing the Certification Regarding Nondiscrimination contained in Exhibit II-6 of the Proposal & Award Policies & Procedures Guide.

Certification Regarding Flood Hazard Insurance

Two sections of the National Flood Insurance Act of 1968 (42 USC §4012a and §4106) bar Federal agencies from giving financial assistance for acquisition or construction purposes in any area identified by the Federal Emergency Management Agency (FEMA) as having special flood hazards unless the:

(1) community in which that area is located participates in the national flood insurance program; and

(2) building (and any related equipment) is covered by adequate flood insurance.

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant located in FEMA-designated special flood hazard areas is certifying that adequate flood insurance has been or will be obtained in the following situations:

(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and

(2) for other NSF grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

Certification Regarding Responsible Conduct of Research (RCR)
(This certification is not applicable to proposals for conferences, symposia, and workshops.)

By electronically signing the Certification Pages, the Authorized Organizational Representative is certifying that, in accordance with the NSF Proposal & Award Policies & Procedures Guide, Chapter IX.B. , the institution has a plan in place to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduates, graduate students and postdoctoral researchers who will be supported by NSF to conduct research. The AOR shall require that the language of this certification be included in any award documents for all subawards at all tiers.

CERTIFICATION PAGE - CONTINUED

Certification Regarding Organizational Support

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that there is organizational support for the proposal as required by Section 526 of the America COMPETES Reauthorization Act of 2010. This support extends to the portion of the proposal developed to satisfy the Broader Impacts Review Criterion as well as the Intellectual Merit Review Criterion, and any additional review criteria specified in the solicitation. Organizational support will be made available, as described in the proposal, in order to address the broader impacts and intellectual merit activities to be undertaken.

Certification Regarding Federal Tax Obligations

When the proposal exceeds $5,000,000, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal tax obligations. By electronically signing the Certification pages, the Authorized Organizational Representative is certifying that, to the best of their knowledge and belief, the proposing organization:
(1) has filed all Federal tax returns required during the three years preceding this certification;
(2) has not been convicted of a criminal offense under the Internal Revenue Code of 1986; and
(3) has not, more than 90 days prior to this certification, been notified of any unpaid Federal tax assessment for which the liability remains unsatisfied, unless the assessment is the subject of an installment agreement or offer in compromise that has been approved by the Internal Revenue Service and is not in default, or the assessment is the subject of a non-frivolous administrative or judicial proceeding.

Certification Regarding Unpaid Federal Tax Liability

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal Tax Liability:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has no unpaid Federal tax liability that has been assessed, for which all judicial and administrative remedies have been exhausted or lapsed, and that is not being paid in a timely manner pursuant to an agreement with the authority responsible for collecting the tax liability.

Certification Regarding Criminal Convictions

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Criminal Convictions:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has not been convicted of a felony criminal violation under any Federal law within the 24 months preceding the date on which the certification is signed.

Certification Dual Use Research of Concern

By electronically signing the certification pages, the Authorized Organizational Representative is certifying that the organization will be or is in compliance with all aspects of the United States Government Policy for Institutional Oversight of Life Sciences Dual Use Research of Concern.

AUTHORIZED ORGANIZATIONAL REPRESENTATIVE		SIGNATURE	DATE
NAME Sonjie Schwartz, Interim Director Office of Sponsored Projects Administration		Sonjie Schwartz	9/17/18
TELEPHONE NUMBER	EMAIL ADDRESS		FAX NUMBER
618-453-4540	ospa@siu.edu		618-453-8038

PROJECT SUMMARY

Overview:

Tensor is a generalization from vector and matrix to represent multi-dimensional datasets. Methods based on tensor formulation have been extensively developed in recent years in order to tackle various complex multi-dimensional problems, and have been successfully applied to numerous areas, such as signal processing, computer vision, machine learning, image processing, data mining, and among others. When appropriately developed via mathematical theory, many tensor methods are becoming approachable by making use of highly optimized linear algebra libraries and can be run on modern systems for large-scale computations.

Low rank approximation of higher-order tensors is highly desirable in various practical applications and is becoming a main theme in order to process multi-dimensional arrays efficiently and effectively. Quite often in many applications, due to some local similarity or certain periodicity of a highly multi-dimensional array such as in image processing, the rank of such a tensor usually appears to be significantly lower than its size. When a multi-dimensional array is governed by a low-rank structure, the handling of such a dataset becomes much more approachable, and more importantly, the low rank property indicates that the dataset can be significantly compressed in a meaningful way. This explains why the low rank characteristic is so attractive and practically useful in various applications.

During the last decade, the low rank approximation of tensors mainly focuses on convex regularization, and the approach appears to be insufficient due to the limitations of convex formulation. In this proposal, we will develop the low rank approximation of tensors via non-convex regularization, which currently is not well established yet for the study of multi-dimensional datasets. There is very few study for tensors under the non-convex formulation at this point. Under the tensor inner product vector space setting, we propose a framework in which equivalent problems can be formulated in the Fourier domain, where tensor ranks can be characterized in a more approachable way. Preliminary experiments for 3rd-order tensor on tensor completion problem, tensor robust principal component analysis, and multi-view clustering, respectively, show that our proposed formulation by using non-convex regularization is quite promising, compared to latest available developments in literature.

Intellectual Merit:

Systematic study of low rank approximation of tensorial data under an inner product tensor space via non-convex formulation has not been available yet. Many current studies, orientated from individual problems, are focused on case by case. Non-convex formulation can provide more effective approach for tensor related problems. To formulate these problems under vector space theory, such as introducing Fourier transform and induced tensor norm, can increase mathematical capacity that lead to better solutions. When formulated appropriately by mathematics, the low rank approximation of tensors can be addressed by the optimization theory in vector space, leading to expanding our research capacity.The research subject and the proposed approach are of theoretical and practical importance. Mathematically, a tensor can be viewed as an element in a vector space and thus it is plausible to believe that non-convex optimization techniques resulted from the theory of optimization in vector space should be applicable for tensor case, and the corresponding theory/method are required to re-examine and further to develop. This proposal proposes new developments of low rank approximation of tensors via non-convex formulation that addresses the essential issues for multi-dimensional dataset.

Broader Impacts:

The proposed research activity (1) promotes the creation and development of the next generation of mathematical theory and tools;(2) advances existing approaches for the tensor low rank approximation by non-convex setting; (3) establishes a new graduate course and enhances the multidisciplinary program at PI's institute, and this project will provide support, mentorship and training for graduate/undergraduate students; (4) promote mathematical learning interests for K-12 students, including to recruit students from Brehm Preparatory School with learning disabilities, in the local community at Southern Illinois.

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For font size and page formatting specifications, see PAPPG section II.B.2.

Appendix Items:

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

Study of Low Rank Approximation of Tensorial Data Set via Non-convex Regularization

MingQing Xiao

1 Introduction

Modern big data analysis requires to handle large amounts of data associated with many variables, which naturally leads to the study of highly multi-dimensional problems. Essentially, the "big" of big data is mainly reflected by two characteristics: large quantity and high dimensions, that brings fundamental challenges to the study of big datasets nowadays.

Tensor is a generalization from vector and matrix to represent multi-dimensional datasets [20]. Given a reference basis of vectors, a tensor can be represented as an organized multidimensional array of numerical values, whose elements can be written as $[x_{i_1 i_2 \cdots i_N}]$. More specifically, an array of numbers appeared on a regular grid with a variable number of axes is described as a tensor. When $N = 1$, it stands for a vector and when $N = 2$, it simply represents a matrix, tensors of order three or higher are called higher-order tensors in literature. Formally, an N-way or Nth-order tensor is an element of the tensor product of N vector spaces, each of which has its own coordinate system.

Methods based on tensor formulation have been extensively developed in recent years in order to tackle various complex multi-dimensional problems, and have been successfully applied to numerous areas, such as signal processing, computer vision, machine learning, pattern recognition, data mining, image procesing, and among others (see survey paper [18] and references therein). When appropriately developed, many tensor methods are becoming approachable by making use of highly optimized linear algebra libraries and can be run on modern systems for large-scale computations.

The structural properties of tensors play fundamental roles in both analyzing and processing various data sets represented by tensors. In particular, the tensor rank is essentially defined to characterize the dimensionality of the array needed to represent it, or equivalently, the number of indices needed to label a component of that array. Quite often in many applications, due to some local similarity or certain periodicity of a highly multi-dimensional array such as in image processing, the rank of such a tensor usually appears to be significantly lower than its size. When a multi-dimensional array is governed by a low-rank structure, the handling of such a dataset becomes much more approachable, and more importantly, the low rank property indicates that the dataset can be significantly compressed in a meaningful way. This explains why the low rank characteristic is so attractive and practically useful in various applications.

Low rank approximation of higher-order tensors is highly desirable in various practical applications and is becoming a main theme in order to process multi-dimensional arrays efficiently and effectively. For instance, in the study of tensor completion, which naturally arises in data-driven applications such as image completion and video compression, the simplest formulation can be described as following.

Suppose that we are given an observed data tensor $\mathcal{M}$ with missing elements. Due to the nature of $\mathcal{M}$, we would like to find a tensor $\mathcal{X}$ of the same size such that $\mathcal{X}_\Omega = \mathcal{M}_\Omega$, where Ω denotes the index of known elements, and the rank of $\mathcal{X}$ is require to be as low as possible. It is ready to see that such a problem can be formulated as the following optimization problem:

$$\min_{\mathcal{X}} : \mathrm{rank}(\mathcal{X}), \;\; \text{subject to } \; \mathcal{X}_\Omega = \mathcal{M}_\Omega.$$

The challenge of above optimization problem is immediately seen since the function "rank$(\cdot)$" is neither convex nor smooth, and standard techniques cannot be applied even if we put the difficulty resulted from the complex structure of tensor itself aside.

In current literature, for a given tensor $\mathcal{M}$, many real-world problems related to its low rank approximation can be cast as the following general type of optimization problem (or sub-problem):

$$\min_{\mathcal{X}\in C} : \Phi(\mathcal{X}-\mathcal{M}) + \lambda\, \Psi(\mathcal{X}), \tag{1.1}$$

where $\Phi(\cdot)$, $\Psi(\cdot)$ are nonnegative functions in appropriate domains, C represents some admissible constraints, and λ is a positive parameter. $\Phi(\cdot)$ usually is a convex function being used to characterize the difference between $\mathcal{X}$ and $\mathcal{M}$, while $\Psi(\cdot)$ is used to 'regulate' some desirable properties, such as requirements of low rank, sparsity, etc.. The parameter λ is used to 'balance' the minimization outcome so that on one hand, the obtained $\mathcal{X}^*$ can be sufficiently close to $\mathcal{M}$, on the other hand, $\Psi(\mathcal{X}^*)$ can retain the desired property as much as it can. Such a formulation has been adopted as the latest framework in scientific computing in order to address various large-scale linear and multilinear algebra problems in current literature (e.g., see survey paper [9] for different individual cases).

In general, the function $\Psi(\cdot)$ is non-convex, e.g., $\Psi(\mathcal{X})$ =rank$(\mathcal{X})$, which leads to the minimization problem (1.1) being practically intractable. To overcome this challenging, most of existing approaches is to replace $\Psi(\cdot)$ by its *lower* convex envelope $\Psi_{env}(\cdot)$, which can be described as

$$\Psi_{env}(\mathcal{X}) = \sup\{g(\mathcal{X}) \mid g \text{ is convex and } g \le \Psi \text{ over an appropriate defined unit ball of } \mathcal{X}\},$$

and instead of solving (1.1), to solve the following convex (thus it is tractable) minimization problem:

$$\min_{\mathcal{X}\in C} : \Phi(\mathcal{X}-\mathcal{M}) + \lambda\, \Psi_{env}(\mathcal{X}), \tag{1.2}$$

where $\Psi_{env}(\cdot)$ can be formally obtained by finding the twice conjugate of $\Psi(\cdot)$, which is the tightest convex relaxation of $\Psi(\cdot)$ from below. Currently, this approach is widely used in the study of low rank approximation (as well as some other similar problems) for high-order tensors. In spite of being tractable, however, this approach suffers from two critical drawbacks: (i) the gap between the non-convex $\Psi(\mathcal{X})$ and its lower convex envelope $\Psi_{env}(\mathcal{X})$ may not be very small, and this approach overpenalizes those larger singular values of a given tensor $\mathcal{X}$ that leads to the tensor rank not being well approximated; (ii) the approach is hardly to be further improved in general under the framework of convex regularization.

In order to illustrate the aforementioned issues in detail, suppose $\Psi(\mathcal{X})$ =rank$_a(\mathcal{X})$, where $\mathcal{X}$ is a 3rd-order tensor and rank$_a(\cdot)$ stands for the tensor average rank. It has been shown that $\Psi_{env}(\mathcal{X}) = \|\mathcal{X}\|_*$ over a unit ball governed by the tensor spectral norm [25], where $\|\cdot\|_*$ is the tensor nuclear norm, i.e., the sum of its singular values. Thus solving the minimization of (1.2) is a process moving in terms of the sum of all singular values, regardless of their difference. As we know, larger singular values usually carry more physical information, while smaller singular values play much less role in the characterization of a tensor

(e.g., by tensor SVD). More specifically, in solving (1.2) by iteration, $\mathcal{X}_{i+1}^* = \mathcal{X}_{i+1}^*(\mathcal{X}_i^*, M, \lambda_i)$ depends on the choice of λ_i which is based only on the information of the linear sum of singular values of $\mathcal{X}_i^*$, and this may lead to inaccurate approximation when some singular values are very large and the rest are quite small. This can be seen already for the matrix cases, confirmed by many known experiments (e.g., see [7]). Some improvements, e.g., by weighted nuclear norm, have been achieved, however, since $\Psi_{env}(\cdot)$ is the *tightest* convex relaxation of $\Psi(\cdot)$ according to convex analysis, a further improvement under convex regularization in general may be very limited, if it is still possible, in particular when the gap between $\Psi(\cdot)$ and $\Psi_{env}(\cdot)$ appears to be not so small and high accuracy approximation is required.

In this proposal, we will develop the low rank approximation of tensors via non-convex regularization, which currently is not well established yet for the study of multi-dimensional datasets. There is very few study *for tensors* under the non-convex formulation at this point. From the viewpoint of optimization, non-convex formulation in general is better in capturing the problem structure than its convex counterpart. Mathematically, a tensor can be viewed as an element in a vector space and thus it is plausible to believe that with the help from geometric interpretation, non-convex optimization techniques resulted from the theory of optimization in vector space should be applicable for tensor case, though it is expected not to be straightforward. The main challenge is how to maintain the tensor structure *adequately* during the optimization process so that no critical inherent information is lost and valid solution can be obtained. Many optimization questions under the framework of tensor formulation remain unanswered yet. Although non-convex optimization problems usually suffer from intractability, however, careful modeling/setting often can circumvent the difficulty, as shown in modern optimization theory for other non-tensor problems.

Technically, in our proposed framework, we first replace the intractable term $\Psi(\cdot)$ in (1.1) by carefully constructing a non-convex but smooth function $\Psi_0(\cdot)$ as a smooth surrogate for tensor rank. Then we use discrete Fourier transform (DFT) to transfer the tensor minimization problem (1.1) to the one in Fourier domain, where an equivalent optimization problem is formulated with more transparent tensor rank characteristics. We then introduce non-convex regularization and solve the problem there. After obtaining the optimal solution in the Fourier domain, it will be transformed back to the original domain by the inverse discrete Fourier transform, which yields the low rank approximation. More specifically, some desirable smooth surrogate function $\Psi_0(\cdot)$ is constructed so that it is unitarily invariant with appropriate inner product defined in the usual tensor (vector) space. Then we use DFT, developed recently by Kilmer and Martin ([17], 2011), to obtain a tensor decomposition in Fourier domain, in which an equivalent and feasible minimization problem is established and solved by optimization theory. Our preliminary study reveals that solving the optimization problem in the Fourier domain can better simplify the tensor inherent structure under our proposed formulation. The proposed idea, to the best of our knowledge, has not been seen in current available research. We have conducted some preliminary experiments for 3rd-order tensor on tensor completion problem, tensor robust principal component analysis, and multi-view clustering, respectively, and the experiment results show that our proposed formulation by using non-convex regularization is quite promising, compared to latest developments, as shown later on in this proposal.

2 Low Rank Approximation via Non-convex Regularization

For the convenience of our description, in this proposal, we mainly focus on 3rd-order tensor for the development of the proposed idea. In principle, higher-order tensor can be handled recursively by lower-order tensor, though the analysis may be subtle but it is a natural development in general.

Notations and Definitions: Following the convention, we use the boldface Euler script letter, e.g., $\mathcal{X}$, to represent a tensor; and use the boldface capital letter, e.g., $\mathbf{X}$, to stand for a matrix, and use lowercase letter, e.g., a to denote a scalar. For a 3rd-order tensor $\mathcal{X} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$, we use $\mathcal{X}(i,:,:)$, $\mathcal{X}(:,i,:)$, $\mathcal{X}(:,:,i)$ to denote the i-th horizontal, lateral, and frontal slice, respectively. The front slice $\mathcal{X}(:,:,i)$ is denoted compactly as $\mathbf{X}_i$. The inner product between $\mathbf{X}$ and $\mathbf{Y}$ in $\mathbb{C}^{n_1 \times n_2}$ is defined as $< \mathbf{X}, \mathbf{Y} >= \mathrm{tr}(\mathbf{X}^* \mathbf{Y})$, where $\mathbf{X}^*$ denotes the conjugate transpose of $\mathbf{X}$. The inner product between $\mathcal{X}$ and $\mathcal{Y}$ in $\mathbb{C}^{n_1 \times n_2 \times n_3}$ is defined as $< \mathcal{X}, \mathcal{Y} >= \sum_{i=1}^{n_3} < \mathbf{X}_i, \mathbf{Y}_i >$. The Frobenius norm of $\mathcal{X} = [x_{ijk}]$ is defined as $\|\mathcal{X}\|_F = \sqrt{< \mathcal{X}, \mathcal{X} >} = \sqrt{\sum_{ijk} |x_{ijk}|^2}$, and the ℓ_1 norm of $\mathcal{X}$ is defined as $\|\mathcal{X}\|_1 = \sum_{ijk} |x_{ijk}|$. The block circulant matrix $\mathrm{bcirc}(\mathcal{X}) \in \mathbb{R}^{n_1 n_3 \times n_2 n_3}$ of $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ generated by its frontal slices is defined as

$$
\mathrm{bcirc}(\mathcal{X}) = \begin{bmatrix} X_1 & X_{n_3} & X_{n_3-1} & \cdots & X_2 \\ X_2 & X_1 & X_{n_3} & \cdots & X_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{n_3} & X_{n_3-1} & \cdots & X_2 & X_1 \end{bmatrix}_{n_1 n_3 \times n_2 n_3}.
$$

It is important to notice that the block circulant matrix $\mathrm{bcirc}(\mathcal{X})$ keeps the order of frontal slices of $\mathcal{X}$ in an appropriate way, and thus it better maintains $\mathcal{X}$'s structure in terms of the frontal direction, compared with the direct tensor unfolding along the third mode. The Discrete Fourier Transform matrix $\mathbf{F}_n$ is defined as

$$
\mathbf{F}_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & w^{n-1} & \cdots & w^{(n-2)(n-1)} & w^{(n-1)(n-1)} \end{bmatrix}_{n \times n} \in \mathbb{C}^{n \times n},
$$

where $w = e^{-\frac{2\pi i}{n}}$ is a primitive n-th root of unity in which $i = \sqrt{-1}$. If we define $\widetilde{\mathbf{F}}_n = \frac{1}{\sqrt{n}} \mathbf{F}_n$, then it can be verified that $\widetilde{\mathbf{F}}_n^* \widetilde{\mathbf{F}}_n = \widetilde{\mathbf{F}}_n^* \widetilde{\mathbf{F}}_n = \mathbf{I}_n$, i.e., $\widetilde{\mathbf{F}}_n$ is an orthogonal matrix. For $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, the vec and fold operators are defined, respectively, as follows:

$$
vec(\mathcal{X}) = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{n_3} \end{bmatrix}_{n_1 n_3 \times n_2} \quad \text{and} \quad fold(vec(\mathcal{X})) = \mathcal{X}.
$$

Let $\mathcal{A} \in \mathbb{R}^{n_1 \times n_4 \times n_3}$, and $\mathcal{B} \in \mathbb{R}^{n_4 \times n_2 \times n_3}$. Then the t-product of 3rd-order tensors is defined as

$$
\mathcal{A} * \mathcal{B} = fold(bcirc(\mathcal{A}) vec(\mathcal{B})) \in \mathbb{R}^{n_1 \times n_2 \times n_3}.
$$

For $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, the transpose tensor $\mathcal{X}^{\mathrm{T}}$ is an $n_2 \times n_1 \times n_3$ tensor obtained by transposing each frontal slice of $\mathcal{X}$ and then reversing the order of the transposed frontal slices 2 through n_3. The identity tensor $\mathcal{I} \in \mathbb{R}^{n_1 \times n_1 \times n_3}$ is a tensor whose first frontal slice is the $n_1 \times n_1$ identity matrix and all other frontal slices are zero. A tensor $\mathcal{Q} \in \mathbb{R}^{n_1 \times n_1 \times n_3}$ is orthogonal if $\mathcal{Q}^{\mathrm{T}} * \mathcal{Q} = \mathcal{Q} * \mathcal{Q}^{\mathrm{T}} = \mathcal{I}$, where $*$ is the t-product. A tensor is called f-diagonal if each of its frontal slices is diagonal matrix. According to Lu et al. ([25], 2018), the tensor Singular Value Decomposition (t-SVD) of $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ can be expressed by

$$
\mathcal{X} = \mathcal{U} * \Sigma * \mathcal{V}^{\mathrm{T}}, \quad \mathcal{U} \in \mathbb{R}^{n_1 \times n_1 \times n_3}, \Sigma \in \mathbb{R}^{n_1 \times n_2 \times n_3}, \mathcal{V} \in \mathbb{R}^{n_2 \times n_2 \times n_3},
$$

where $\mathcal{U}$ and $\mathcal{V}$ are orthogonal tensors, respectively, and Σ is an f-diagonal tensor, and $*$ denotes the t-product. The tensor *tubal rank* (see [17]) $\text{rank}_t(\mathcal{X})$ is defined as the number of nonzero singular tubes of f-diagonal tensor Σ, which is the maximum of the number of nonzero $\{\Sigma_{i,i,i_3},\ 1 \le i \le \min(n_1,n_2)\}$ for $1 \le i_3 \le n_3$., and the *average-rank* (see [25]) $\text{rank}_a(\mathcal{X})$ of $\mathcal{X}$ is defined as $\text{rank}_a(\mathcal{X}) = 1/n_3\ \text{rank}(\text{bcirc}(\mathcal{X}))$. Both tensor tubal rank and tensor average rank appear not only to be natural generalization of matrix rank but also to be tractable (through Fourier transform shown next), comparing to CP rank, defined as the smallest number of rank-one tensor decomposition, which is generally NP-hard to compute. Other tractable rank definitions such as Tucker rank [18] and tensor multi-rank [16] can be essentially incorporated into our following proposed approach with some extra but workable treatment.

Fast Fourier Transform: The block circulant matrix $\text{bcirc}(\mathcal{X})$ can be block-diagonalized as

$$\left(\widetilde{\mathbf{F}}_{n_3} \otimes \mathbf{I}_{n_1}\right) \cdot \text{bcirc}(\mathcal{X}) \cdot \left(\widetilde{\mathbf{F}}^*_{n_3} \otimes \mathbf{I}_{n_2}\right) = \begin{bmatrix} \hat{\mathbf{X}}_1 & & \\ & \ddots & \\ & & \hat{\mathbf{X}}_{n_3} \end{bmatrix}_{n_1 n_3 \times n_2 n_3} := \hat{\mathbf{X}},$$

and if we use $\hat{\mathbf{X}}_1, \cdots, \hat{\mathbf{X}}_{n_3}$ as frontal slices to construct a tensor $\hat{\mathcal{X}}$ in $\mathbb{C}^{n_1 \times n_2 \times n_3}$, then $\hat{\mathcal{X}}$ is viewed as the tensor $\mathcal{X}$ in Fourier domain. It can be verified that $\|\mathcal{X}\|_F = \|\hat{\mathcal{X}}\|_F$. For each matrix $\hat{\mathbf{X}}_{i_3}, 1 \le i_3 \le n_3$, we denote its SVD as $\hat{\mathbf{X}}_{i_3} = \hat{\mathbf{U}}_{i_3} \hat{\Sigma}_{i_3} \hat{\mathbf{U}}^*_{i_3}$, then we further have

$$\begin{bmatrix} \hat{\mathbf{X}}_1 & & \\ & \ddots & \\ & & \hat{\mathbf{X}}_{n_3} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{U}}_1 & & \\ & \ddots & \\ & & \hat{\mathbf{U}}_{n_3} \end{bmatrix} \cdot \begin{bmatrix} \hat{\Sigma}_1 & & \\ & \ddots & \\ & & \hat{\Sigma}_{n_3} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}^*_1 & & \\ & \ddots & \\ & & \hat{\mathbf{V}}^*_{n_3} \end{bmatrix}.$$

If we use the diagonal blocks of above each block matrix as the frontal slices to recover the corresponding tensor, respectively, we have $\hat{\mathcal{X}} = \hat{\mathcal{U}} * \hat{\Sigma} * \hat{\mathcal{V}}^*$, an t-SVD of $\mathcal{X}$ in the Fourier domain. It is not difficult to verify that $\mathcal{X} \to \hat{\mathcal{X}}$ is an one-to-one mapping, preserving the Frobenius norm. One of advantages by introducing Fourier transform under above framework is to allow us to simplify the t-SVD decomposition since a *direct* decomposition $\mathcal{X} = \mathcal{U} * \Sigma * \mathcal{V}^T$ usually is a formidable task. By using the properties of circulant matrix, a simple algorithm can guarantee the inverse Fourier transform of $\hat{\mathcal{U}} \to \mathcal{U}$, $\hat{\mathcal{V}} \to \mathcal{V}$ to be tensors with *real* entries [25]. More importantly, the above t-SVD is consistent with the matrix SVD and provides the best tensor rank approximation characteristics under the tensor Fobenius norm [25].

Formulation on the Fourier Domain: Instead of considering (1.1), we study the following problem

$$\min_{\hat{\mathcal{X}} \in \hat{\mathcal{C}}} : \Phi(\hat{\mathcal{X}} - \hat{\mathcal{M}}) + \lambda\, \Psi(\hat{\mathcal{X}}), \tag{2.3}$$

in the Fourier domain, which appears to be more approachable than problem (1.1). To see that, suppose that $\Phi(\hat{\mathcal{X}} - \hat{\mathcal{M}}) = \|\hat{\mathcal{X}} - \hat{\mathcal{M}}\|_F^2$ and $\Psi(\hat{\mathcal{X}}) = \text{rank}_t(\hat{\mathcal{X}})$, then we have

$$\Phi(\hat{\mathcal{X}} - \hat{\mathcal{M}}) + \lambda \Psi(\hat{\mathcal{X}}) = \|\hat{\mathcal{X}} - \hat{\mathcal{M}}\|_F^2 + \lambda\, \text{rank}_t(\hat{\mathcal{X}}) = \sum_{i_3=1}^{n_3} \left(\|\hat{\mathbf{X}}_{i_3} - \hat{\mathbf{M}}_{i_3}\|_F^2 + \lambda\, \text{rank}(\hat{\mathbf{X}}_{i_3}) \right),$$

which is reduced to a *matrix* problem for a 3rd-order tensor problem. In order to maintain the characteristics of the original problem (1.1), we require that the optimal solution under the Fourier transform should be invariant, that is,

$$\mathcal{X}^* = \arg\min_{\mathcal{X} \in \mathcal{C}} : \Phi(\mathcal{X} - \mathcal{M}) + \lambda\, \Psi(\mathcal{X}) \ \leftrightarrow\ \hat{\mathcal{X}}^* = \arg\min_{\hat{\mathcal{X}} \in \hat{\mathcal{C}}} : \Phi(\hat{\mathcal{X}} - \hat{\mathcal{M}}) + \lambda \Psi(\hat{\mathcal{X}}).$$

But this usually generate little problem due to the natural formulations of Φ and Ψ. For example, again let $\Phi(\mathcal{X}-\mathcal{M})=\|\mathcal{X}-\mathcal{M}\|_F^2$ and $\Psi(\mathcal{X})=\text{rank}_t(\mathcal{X})$, if $\mathcal{X}\leftrightarrow\hat{\mathcal{X}}$, $\mathcal{M}\leftrightarrow\hat{\mathcal{M}}$ then we have

$$\Phi(\mathcal{X}-\mathcal{M})=\|\mathcal{X}-\mathcal{M}\|_F^2=\|\hat{\mathcal{X}}-\hat{\mathcal{M}}\|_F^2=\Phi(\hat{\mathcal{X}}-\hat{\mathcal{M}}) \text{ and } \Psi(\mathcal{X})=\text{rank}_t(\mathcal{X})=\text{rank}_t(\hat{\mathcal{X}})=\Psi(\hat{\mathcal{X}})$$

which guarantees that $\mathcal{X}^*\leftrightarrow\hat{\mathcal{X}}^*$. This property allows us to establish an equivalent problem of (1.1) in the Fourier domain, formulating the original problem in a more approachable way.

Non-convex Regularization: There are many existing studies on the non-convex rank approximation for *matrices* (for example, see [7, 6, 45, 33, 32, 28, 14, 10]). As a starting point, for a matrix $\mathbf{X}\in\mathbb{R}^{m\times n}$, we consider two common smooth, non-convex surrogates for rank($\mathbf{X}$) in literature. The first one is the ε-log-determinant of $\mathbf{X}$ is given by $\Psi_0(\mathbf{X})=\log\det(\varepsilon I+(\mathbf{X}^T\mathbf{X})^{1/2})$, provided that $\varepsilon\geq 0$, $\varepsilon+\sigma_i(\mathbf{X})\geq 1$, and it is easy to see that $\Psi_0(\mathbf{X})=\sum_{i=1}^{\min(m,n)}\log(\varepsilon+\sigma_i(\mathbf{X}))$ and is smooth. In many cases, we set $\varepsilon=1$ if there are some $\sigma_i(\mathbf{X})=0$. Another one is to use the Schatten-p norm $(0<p<1)$ of X, which is a pre-norm and defined as

$$\|\mathbf{X}\|_{S_p}=\left(\sum_{i=1}^{\min\{m,n\}}\sigma_i^p(\mathbf{X})\right)^{1/p}.$$

Both non-convex functions show their effectiveness in many *matrix* related problems. From the mathematical point of view, both non-convex regularization share the same characteristic: it is a function of singular values of $\mathbf{X}$ and is unitarily invariant, which implies that the techniques are essentially built on the matrix singular decomposition. Thus tensor approach should carry the same fundamental. For our proposed approach, for example, since $\text{rank}_a(\mathcal{X})=\text{rank}_a(\hat{\mathcal{X}})=\text{rank}_a(\hat{\Sigma})$, by making use of the unitarily invariant, we construct Ψ_0 for the ε-log-determinant of $\mathcal{X}\to\hat{\mathcal{X}}=\hat{\mathcal{U}}*\hat{\Sigma}*\hat{\mathcal{V}}^\Gamma\in\mathbb{R}^{n_1\times n_2\times n_3}$ directly in the Fourier domain as

$$\Psi_0(\hat{\mathcal{X}})=\frac{1}{n_3}\log\det\left(\text{bdiag}(\mathcal{E})+\text{bdiag}(\hat{\Sigma})\right)=\frac{1}{n_3}\sum_{i_3=1}^{n_3}\sum_{i=1}^{\min\{n_1,n_2\}}\log\left(\varepsilon_{i_3}+\hat{\Sigma}_{i,i,i_3}\right), \tag{2.4}$$

where $\mathcal{E}\in\mathbb{R}^{n_1\times n_2\times n_3}$ with its frontal slice $\mathcal{E}_{i_3}=\varepsilon_{i_3}I_{n_1\times n_2}$, where $I_{n_1\times n_2}$ is a diagonal matrix with main diagonal entries being 1, $\varepsilon_{i_3}\geq 0$ is a scalar. The notation 'bdiag($\cdot$)' denotes the block diagonal matrix consisting of frontal slices of a tensor. For the simplicity, we sometimes use the following non-convex regularization

$$\Psi_0(\hat{\mathcal{X}})=\frac{1}{n_3}\sum_{i_3=1}^{n_3}\sum_{i=1}^{\min\{n_1,n_2\}}\log\left(1+\hat{\Sigma}_{i,i,i_3}^2\right). \tag{2.5}$$

Similarly, for the Schatten-p norm case, Ψ_0 can be constructed in the Fourier domain as

$$\Psi_0(\hat{\mathcal{X}})=\frac{1}{n_3}\sum_{i_3=1}^{n_3}\left(\sum_{i=1}^{\min\{n_1,n_2\}}\hat{\Sigma}_{i,i,i_3}^p\right)^{1/p}, \tag{2.6}$$

Notice that both constructions are consistent with matrix case when $n_3=1$. Throughout this proposal, we call (2.4), (2.5), and (2.6) the t-ε-LogDet, t-Logdet, and t-S_p regularization, respectively. The difference between LogDet and t-S_p is that t-S_p is a pre-norm, while LogDet is not. Due to the variety of different problems, each problem itself may permit some 'good' non-convex regularization Ψ_0, in addition to LogDet and t-S_p. The construction of Ψ_0 for low rank approximation may depend on (i) the distribution of singular values; (ii) the inherent constraints resulted from the problem itself. The first one is easy

to understand since we would like to retain those large singular values and to discard those very small singular values, a fundamental idea in rank approximation. Thus tensor SVD plays important role in this regard and Ψ_0 should be unitarily invariant in $\mathbb{R}^{n_1 \times n_2 \times n_3}$ with a well-defined inner product. The second one is quite subtle. For example, in the image recovery of tensor robust PCA, the non-convex regularization t-S_p $(0 < p < 1)$ seems working better than LogDet. Moreover, once the problem is formulated via Lagrange multiplier, the choice of Ψ_0 will affect the convergence of a KKT (Karush-Kuhn-Tucker) point of the problem. These issues, to our point of view, can be addressed under the framework of optimization theory in vector space. More specifically, we will study the Karush-Kuhn-Tucker local theory in tesnsor space under the proposed formulation. Many matrix theories, such as matrix induced norm, spectral radius of matrices, as well as Von-Neumann's trace inequality, are required to be re-examined and developed for tensor case, due to the incomplete development in current literature.

Next we illustrate how our general formulation (2.3) can be used in various applications.

A. Tensor competition: Given an observed tensor $\mathcal{M}$ of size $n_1 \times n_2 \times n_3$ whose entries in the index set $\{M_{ijk} : (i,j,k) \in \Omega\}$, where Ω is an indicator tensor of size $n_1 \times n_2 \times n_3$, are known but the remaining are missing, find the tensor $\mathcal{X}$ of size $n_1 \times n_2 \times n_3$ from the following optimization problem to recover the entire $\mathcal{M}$ with a non-convex function Ψ_0 that characterizes the low rank property:

$$\min : \Psi_0(\mathcal{X}) \text{ subject to } P_\Omega(\mathcal{X}) = P_\Omega(\mathcal{M}) \tag{2.7}$$

where P_Ω is the orthogonal projector onto the span of tensors vanishing outside of Ω. So the $(i,j,k)_{th}$ component of $P_\Omega(\mathcal{X})$ is equal to $\mathcal{M}_{ijk}$ if $(i,j,k) \in \Omega$ and zero otherwise. Let $\mathcal{Y}$ be the available (sampled) data: $\mathcal{Y} = P_\Omega \mathcal{M}$. Define $\mathcal{G} = \widetilde{\mathbf{F}}_3 P_\Omega \widetilde{\mathbf{F}}_3^{-1}$ where $\widetilde{\mathbf{F}}_3$ and $\widetilde{\mathbf{F}}_3^{-1}$ are the operators representing the Fourier and inverse Fourier transform along the third dimension of tensors. Then we have $\hat{\mathcal{Y}} = \mathcal{G}(\hat{\mathcal{M}})$ where $\hat{\mathcal{Y}}$ and $\hat{\mathcal{M}}$ are the Fourier transforms of $\mathcal{Y}$ and $\mathcal{M}$ along the third mode. Thus (2.7) is equivalent to the following, provided that Ψ_0 is unitarily invariant in its tensor (vector) space, as

$$\min : \Psi_0(\hat{\mathcal{X}}) \quad \text{subject to} \quad \hat{\mathcal{Y}} = \mathcal{G}(\hat{\mathcal{X}}). \tag{2.8}$$

To solve the optimization problem, one can re-write (2.8) equivalently as:

$$\min : \Psi_0(\hat{\mathcal{Z}}) + \mathbf{1}_{\hat{\mathcal{Y}} = \mathcal{G}(\hat{\mathcal{X}})} \qquad \text{subject to} \quad \hat{\mathcal{X}} - \hat{\mathcal{Z}} = 0 \tag{2.9}$$

where $\mathbf{1}$ denotes the indicator function. Now we present the numerical scheme for solving Eq. (2.8). By using the general framework of Alternating Direction Method of Multipliers(ADMM) for non-convex problems (see chapter 9 of [2] or [34]), the augumented Lagrangian is set to be

$$\Psi_0(\hat{\mathcal{Z}}) + \mathbf{1}_{\hat{\mathcal{Y}} = \mathcal{G}(\hat{\mathcal{X}})} + \langle \hat{\mathcal{Q}}, \hat{\mathcal{X}} - \hat{\mathcal{Z}} \rangle + \frac{\rho}{2} ||\hat{\mathcal{X}} - \hat{\mathcal{Z}}||_F^2,$$

where $\hat{\mathcal{Q}}$ is the Lagrangian multiplier, ρ is the penalty parameter for the violation of the linear constraints, so that the scaled form of ADMM for this problem is

$$\begin{aligned}
\hat{\mathcal{X}}^{k+1} &= \arg\min_{\hat{\mathcal{X}}} \left\{ \mathbf{1}_{\hat{\mathcal{Y}} = P_\Omega(\hat{\mathcal{X}})} + \hat{\mathcal{X}}^T(:)\hat{\mathcal{Q}}^k(:) + \frac{\rho}{2}\|\hat{\mathcal{X}} - \hat{\mathcal{Z}}^k\|_F^2 \right\} = \arg \min_{\hat{\mathcal{X}}:\hat{\mathcal{Y}} = P_\Omega(\hat{\mathcal{X}})} \left\{ \|\hat{\mathcal{X}} - (\hat{\mathcal{Z}}^k - \frac{1}{\rho}\hat{\mathcal{Q}}^k)\|_F^2 \right\} \\
\hat{\mathcal{Z}}^{k+1} &= \arg\min_{\hat{\mathcal{Z}}} \left\{ \frac{1}{\rho}\Psi_0(\hat{\mathcal{Z}}) + \frac{1}{2}||\hat{\mathcal{Z}} - (\hat{\mathcal{X}}^{k+1} + \frac{1}{\rho}\hat{\mathcal{Q}}^k)||_F^2 \right\} \\
\hat{\mathcal{Q}}^{k+1} &= \hat{\mathcal{Q}}^k + \rho\left(\hat{\mathcal{X}}^{k+1} - \hat{\mathcal{Z}}^{k+1}\right).
\end{aligned}$$

The $\hat{X}(:)$ and $\hat{Q}^k(:)$ means vectorizing the tensors which is also Matlab notation. The first equation is the least-squares projection onto the constraint. The solution is given by $\hat{\mathcal{Z}} = \hat{\mathcal{U}}_{\hat{X}^{k+1}+\hat{Q}^k} * \hat{\Omega}_{\hat{\mathcal{Z}}} * \hat{\mathcal{V}}_{\hat{X}^{k+1}+\hat{Q}^k} \to \mathcal{Z}$, which is recovered by the inverse Fourier transformation.

B. Tensor Robust PCA: Tensor Robust PCA [44, 25](TRPCA) problem aims to recover the low rank component $\mathcal{L}_0$ and sparse component $\mathcal{E}_0$ from noisy observations $\mathcal{O} = \mathcal{L}_0 + \mathcal{E}_0$ of size $n_1 \times n_2 \times n_3$

$$\min_{\mathcal{L},\mathcal{E}} : \Psi_0(\mathcal{L}) + \lambda\|\mathcal{E}\|_1 \quad \text{s.t.} \quad \mathcal{O} = \mathcal{L} + \mathcal{E}. \tag{2.10}$$

It is an elegant extension of Robust PCA [35] for matrix case. Without loss of generality, we assume that all tensors involved here are in Fourier domain. Its Lagrangian function is given by

$$\begin{aligned} L_\beta(\mathcal{L},\mathcal{E},\mathcal{M}) &= \Psi_0(\mathcal{L}) + \lambda\|\mathcal{E}\|_1 + \langle \mathcal{M}, \mathcal{O} - \mathcal{L} - \mathcal{E}\rangle + \frac{\beta}{2}\|\mathcal{O} - \mathcal{L} - \mathcal{E}\|_F^2 \\ &= \Psi_0(\mathcal{L}) + \lambda\|\mathcal{E}\|_1 + \frac{\beta}{2}\|\mathcal{O} - \mathcal{L} - \mathcal{E} - \frac{\mathcal{M}}{\beta}\|_F^2 + C, \end{aligned} \tag{2.11}$$

where $\mathcal{M}$ is the Lagrangian multiplier, β is the penalty parameter for the violation of the linear constraints, and C is a constant. Then, the problem $\arg\min_{\mathcal{L},\mathcal{E},\mathcal{M}} L_\beta(\mathcal{L},\mathcal{E},\mathcal{M})$ in (2.11) can be updated as:

$$\begin{cases} \mathcal{L}^{k+1} = \arg\min\limits_{\mathcal{L}} \|\mathcal{O} - \mathcal{E}^k - \dfrac{\mathcal{M}^k}{\beta} - \mathcal{L}\|_F^2 + \dfrac{2}{\beta}\Psi_0(\mathcal{L}), \\ \mathcal{E}^{k+1} = \mathcal{S}_{\frac{\lambda}{\beta}}\left(\mathcal{O} - \mathcal{L}^{k+1} + \dfrac{\mathcal{M}^k}{\beta}\right), \\ \mathcal{M}^{k+1} = \mathcal{M}^k + \beta(\mathcal{O} - \mathcal{L}^{k+1} - \mathcal{E}^{k+1}), \end{cases} \tag{2.12}$$

where the tensor non-negative **soft-thresholding operator** $\mathcal{S}_\nu(\cdot)$ (similar to [4]) is defined as $\mathcal{S}_\nu(\mathcal{B}) = \bar{\mathcal{B}}$ with

$$\bar{b}_{i_1 i_2 i_3} = \begin{cases} sgn(b_{i_1 i_2 i_3})(|b_{i_1 i_2 i_3}| - \nu), & |b_{i_1 i_2 i_3}| > \nu, \\ 0, & |b_{i_1 i_2 i_3}| \le \nu. \end{cases}$$

The obtained solution is then transformed back to the original domain.

C. Multi-view clustering: Direct unfolding the data or mapping the data to a vector in subspace clustering usually fails to capture the inherent information for muti-dimensional dataset. Recently, in [15, 39, 42, 37], the authors grouped the data into a tensor and proposed a t-SVD based tensor low-rank subspace model for clustering slices of the tensor from multi-view features. It makes use of the subspace representations of different view jointly and explore the intrinsic correlations across different views with clear physical meaning. Let's use $X^{(\nu)}$ to denote the feature matrix corresponding to the ν-th view, and use $Z^{(\nu)}$ to represent the ν-view's of the learned subspace representation. And we will have multiple self-representation coefficient matrix $Z^{(1)}, Z^{(2)}, \ldots, Z^{(V)}$ correspondingly. We keep the low rank constraint for each $Z^{(\nu)}$ and ensure the consensus principle by imposing low rank across all views. Different from [39, 42], by imposing the LogDet tensor rank as given in previous section, we model these two level low rank constraints in a unified tensor space. All the $Z^{(1)}, Z^{(2)}, \ldots, Z^{(V)}$ eventually become close to the desirable structures and the fused $Z = \frac{1}{V}\sum_{\nu=1}^{V}(Z^{(\nu)} + Z^{(\nu)T})$ can be easily segmented by common spectral clustering method. The objective function of the proposed method is the following non-convex optimization problem:

$$\begin{aligned} &\min_{Z^{(\nu)},E^{(\nu)}} \lambda\|E\|_{2,1} + \Psi_0(\mathcal{Z}), \quad s.t. \quad X^{(\nu)} = X^{(\nu)}Z^{(\nu)} + E^{(\nu)},\ \nu = 1,\ldots,V, \\ &\mathcal{Z} = \Phi\left(Z^{(1)}, Z^{(2)}, \ldots, Z^{(V)}\right), E = \left[E^{(1)}; E^{(2)}; \ldots; E^{(V)}\right], \end{aligned} \tag{2.13}$$

where $\|E\|_{2,1}$ is defined by $\|E\|_{2,1} \triangleq \sum_{v=1}^{V} \|(\|\alpha_v^1\|_2, \ldots, \|\alpha_v^N\|_2)\|_1$, and $\alpha_v^i = [\alpha_{i,1}^{(v)}, \ldots, \alpha_{i,d}^{(v)}]^T$ denotes the i^{th} column of matrix $E^{(v)}$. The function $\Phi(\cdot)$ constructs the tensor $\mathcal{Z}$ by merging different representation $Z^{(v)}$ to a 3-mode tensor, and then rotate its dimensionality to $N \times V \times N$. Moreover, one can easily get the following relationship: $\Phi_{(v)}^{-1}(\mathcal{Z}) = Z^{(v)}$, where $\Phi^{-1}(\cdot)$ denotes the inverse function of $\Phi(\cdot)$, and its subscript (v) means to extract the v-th frontal slice. As in [43, 39], $E = \left[E^{(1)}; E^{(2)}; \ldots; E^{(V)}\right]$ is the vertical concatenation along the column of error matrix, which can enforce the column of $E^{(v)}$ in each view to have jointly consistent magnitude values. We then find the optimal self-representations of (2.13) in its Fourier domain. Due to the page limitation, all detailed mathematical discussions, including tensor competition and tensor robust PCA, can be found in our current working manuscripts via https://math.siu.edu/_common/documents/Xiao/filename, where file name can be one of the following: 't.svd.nonconvex.tensor.completion.pdf', 'multiview.clustering.logdet.rank.pdf','Image-Processing.pdf'.

3 Our Preliminary Experiments with Non-convex Regularization

In this section, we present some preliminary experiments and compare them with the latest available resuts. All experiments are implemented in Matlab 2017b on Ubuntu 16.04 LTS with 2.40 GHz CPU, 12 Cores and 64GB RAM. The codes for those existing algorithms we compared with either obtained from individual authors or from public domain where the authors posted. For our approach, all tensors are transformed to the Fourier domain where the non-convex optimization is conducted, as we proposed. The final results are obtained by the inverse Fourier transform. The preliminary experiments show that our proposed framework is quite promising. Here the 'preliminary' means that we only tested t-LogDet, t-ε-LogDet, t-Sp(p=0.4) three non-convex regularization at this point, and the efficiency of our algorithms hasn't been fully optimized yet. Other different non-convex regularization may be possible.

A. Tensor competition: we perform some preliminary experiments on four real-world datasets and compare with the *latest* High accuracy Low Rank Tensor Completion (HaLRTC) [22](2013), Tensor Nuclear Norm(TNN) based approach [44](2014), partial sum of the tensor nuclear (PSTNN) method [13](2017) and weighted logdet based method recently proposed by Teng-Yu Ji(TYJ-LogDet) [12](2017). For each dataset, we consider it as a third order tensor and randomly sample the entries from it. The sampling rate(SR) is set to be 20%, 40% and 60%, respectively. Also, we employ the standard peak signal-to-noise ratio (PSNR) to evaluate the performance of each approach, where PSNR is defined as

$$\mathrm{PSNR}(\mathcal{X}, \tilde{\mathcal{X}}) = 10 \log_{10} \frac{n^2 \mathcal{X}_{max}^2}{\|\mathcal{X} - \tilde{\mathcal{X}}\|_F^2},$$

where $\mathcal{X}, \mathcal{X}_{max}, \tilde{\mathcal{X}}, n$ are origin tensor, maximum pixel value of original tensor, sampling tensor and number of pixels in original tensor, respectively. Higher PSNR usually indicates a better performance. The size of all the datasets are summarized in Table 1. We present the completion results in Table 2-4 and highlight the best performance in boldface. One of three non-convex approaches always appears to be better than others. It is worth mentioning here that a recent approach by Teng-Yu Ji(TYJ-LogDet) [12](2017) directly uses LogDet regularization without transforming the problem to the Fourier domain, and the problem setting there cannot distinguish the order of the frontal slices. Our experiments demonstrate the importance in conducting the optimization in Fourier domain instead of in the original domain, which is essential in our formulation as well as the proposed approach.

B. Tensor robust PCA. We implement the tensor RPCA methods on color images recovery. Follow [13], we perform the algorithms on four high quality color images which come from the Kodak PhotoCD

Table 1: Size of Dataset

Dataset	Size	Data Source
MRI	$181 \times 217 \times 40$	http://brainweb.bic.mni.mcgill.ca/brainweb/selection_normal.html
MSI	$256 \times 256 \times 31$	http://www1.cs.columbia.edu/CAVE/databases/multispectral
Video(Road)	$158 \times 238 \times 24$	http://www.changedetection.net
Video(Basketball)	$144 \times 256 \times 40$	Source: Youtube

Table 2: Completion results with respect to PSNR: sampling rate = 20%

Dataset	Sampled	HaLRTC	TNN	PSTNN	TYJ-LogDet	t-LogDet	t-ε-LogDet	t-Sp(p=0.4)
MRI	10.3989	22.9416	27.5757	28.0169	28.3413	27.3646	**28.8581**	26.2025
MSI	13.8113	23.723	30.1904	31.0145	31.0117	30.9855	**31.5837**	29.1742
Video(Road)	7.1435	22.6751	26.6512	26.9364	27.0692	25.5982	**27.3909**	24.4878
Video(Basketball)	6.2343	17.8314	21.5104	21.3992	21.999	19.4322	**22.0197**	19.2512

Dataset(http://r0k.us/graphics/kodak) and the homepage (https://github.com/canyilu/LibADMM) of the author of [23]. All images are corrupted by the sparse noise with sparsity 0.2, that is, we randomly set 20% of pixels to random values in $[0, 255]$. The last three images are cropped into size 256×256. We compare our approach for ($p = 0.8$) with several other methods: sum of nuclear norms of the unfolding matrices(SNN) [8](2013) based TRPCA method, TNN based TRPCA method [23, 25](2016, 2018), and PSTNN based TRPCA method [13](2017). From the image recovery results (no shown here) we can visually conclude that the proposed method perform better than SNN, TNN and PSTNN methods on all the tested color images. In Table 5, it can be seen that the proposed t-S_p method yields the best result with respect to PSNR which illustrates the merit of our proposed non-convex approach. We haven't tested LogDet yet, thus there is no report here.

C. Multi-view clustering. We perform the proposed approach on some benchmark image clustering datasets[5, 21] and compare our results with six existing methods by running them 20 times and reporting the average performance, following common practice.

1. **Datasets:**
   - **Yale** [1]The Yale face dataset contains 165 grayscale images of 15 individuals. There are 11 images per subject, one per different facial expression or configuration.
   - **Extended YaleB** [2] The Extended YaleB dataset consists of 38 individuals and around 64 near frontal images under different illuminations for each individual. Similarly to the other work, we use the images for the 10 classes, including 640 frontal face images.
   - **ORL** [3] There are 10 different images of each of 40 distinct subjects in the ORL face dataset. The images were took at different times, changing the lighting, facial expressions and facial details for some subjects.
   - **Scene-15** [4] The Scene-15 dataset was gradually built by the works with 15 categories, including office, kitchen, living room, bedroom, etc. Images are about 250×300 resolution, with 210

[1]https://cvc.yale.edu/projects/yalefaces/yalefaces.html
[2]https://cvc.yale.edu/projects/yalefacesB/yalefacesB.html
[3]http://www.uk.research.att.com/facedatabase.html
[4]http://www-cvr.ai.uiuc.edu/ponce grp/data/

Table 3: Completion results with respect to PSNR: sampling rate = 40%

Dataset	Sampled	HaLRTC	TNN	PSTNN	TYJ-LogDet	t-LogDet	t-ε-LogDet	t-Sp(p=0.4)
MRI	11.6423	30.3452	32.9567	33.6019	33.5645	**34.1673**	34.0917	31.5865
MSI	15.0549	27.6852	35.5117	36.2555	35.9875	**36.8898**	36.3877	33.5128
Video(Road)	8.3843	26.766	30.3633	30.6354	30.6412	29.8443	**30.9753**	27.7187
Video(Basketball)	7.4828	21.7232	25.5381	25.9129	26.0616	23.9888	**26.4063**	23.1895

Table 4: Completion results with respect to PSNR: sampling rate = 60%

Dataset	Sampled	HaLRTC	TNN	PSTNN	TYJ-LogDet	t-LogDet	t-ε-LogDet	t-Sp(p=0.4)
MRI	13.3936	35.9134	38.0412	38.4772	38.4817	**39.6931**	38.8049	36.726
MSI	16.8128	31.5046	39.9628	40.4062	40.2508	**41.4351**	40.4929	37.4782
Video(Road)	10.1662	30.4259	33.9881	34.1889	34.1883	34.3369	**34.4584**	31.5282
Video(Basketball)	9.2491	25.3475	29.5397	29.8246	29.9484	28.9392	**30.3903**	27.4137

to 410 image per category. This dataset contains a wide range of outdoor and indoor scene environments.

By adapting feature extraction strategies used in [39], we extract different features based on different applications. The first three datasets are used for face clustering and three types of features are extracted: intensity, LBP[29] and Gabor[19]. The LBP feature is extracted from an image with the sampling size of 8 pixels and the blocking number of 7×8. The Gabor feature is extracted with only one scale $\lambda = 4$ at four orientation $\theta = \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$. Hence, the dimension of LBP feature and Gabor feature is 3304 and 6750 respectively. As for Scene-15, three types of features are extracted: Pyramid histograms of visual words(PHOW)[1], Pairwise rotation invariant co-occurrence local binary pattern feature(PRI-CoLBP)[30] and CENsus TRansform hISTogram(CENTRIST)[36]. PHOW is extracted with dense sampling step of 8 pixels and 300 words and is of 1800 dimensions. PRI-CoLBP is of 1180 dimensions. Three level features are extracted to form CENTRIST, which contains 1, 5, 25 bocks respectively. CENTRIST is of 1240 dimensions.

2. **Existing approaches and our proposed one:**
   - **$\mathbf{SPC_{best}}$[27](2001)** The method employs the most informative view with standard spectral clustering algorithm.
   - **$\mathbf{LRR_{best}}$[21](2013)** Low-rank constraint and the best performed single view feature are used in the method LRR.
   - **RMSC[38](2014)** The method recovers a shared low-rank transition probability matrix for clustering.
   - **DiMSC[3](2015)** Diversity-induced multi-view subspace clustering.
   - **LTMSC[43](2015)** Low-rank tensor constrained multi-view subspace clustering.
   - **t-SVD-MSC[39](2018)** Unifying multi-view self-representations for clustering by tensor multi-rank minimization.

Table 5: Image recovery results with respect to PSNR

Image	Size	Sampled	SNN	TNN	PSTNN	t-Sp(p=0.8, non-convex)
starfish	$481 \times 321 \times 3$	14.8471	24.7843	26.4809	28.9184	**29.6828**
door	$256 \times 256 \times 3$	14.8801	28.4565	31.3789	33.3303	**33.5492**
hat1	$256 \times 256 \times 3$	15.6544	24.7993	26.2922	28.6919	**29.2425**
hat2	$256 \times 256 \times 3$	15.3755	29.323	31.3874	32.0258	**33.7155**

- **LogDet-MSC (this proposal)** In the approach, we transfer the mode-1 fiber coefficient tensor into the mode-3 fiber by using the rotation operation before we transform it to the Fourier domain.

Clustering performance can be evaluated by six popular metrics[26, 11]: Normalized Mutual Information(NMI), Accuracy(ACC), Adjusted Rand index(AR), F-score, Precision and Recall. For all these metrics, higher value indicates better clustering performance. These metrics endorse different properties in the clustering and a comprehensive evaluation can be achieved by various criteria. We present the detailed clustering results and highlight the best performance in below tables.

Table 6: Clustering results(mean) on Yale. We set $\lambda = 0.93$ in proposed *LogDet-MSC*.

Method	NMI	ACC	AR	F-score	Precision	Recall
SPC_{best}	0.654	0.618	0.440	0.475	0.457	0.500
LRR_{best}	0.709	0.697	0.512	0.547	0.529	0.567
RMSC	0.684	0.642	0.485	0.517	0.500	0.535
DiMSC	0.727	0.709	0.535	0.564	0.543	0.586
LTMSC	0.765	0.741	0.570	0.598	0.569	0.629
t-SVD-MSC	0.953	0.963	0.910	0.915	0.904	0.927
LogDet-MSC	**0.985**	**0.978**	**0.969**	**0.964**	**0.971**	**0.978**

Table 7: Clustering results(mean) on Extended YaleB. We set $\lambda = 0.008$ in proposed *LogDet-MSC*.

Method	NMI	ACC	AR	F-score	Precision	Recall
SPC_{best}	0.360	0.366	0.225	0.308	0.296	0.310
LRR_{best}	0.627	0.615	0.451	0.508	0.418	0.539
RMSC	0.157	0.210	0.060	0.155	0.151	0.159
DiMSC	0.636	0.615	0.453	0.504	0.481	0.534
LTMSC	0.637	0.626	0.459	0.521	0.485	0.539
t-SVD-MSC	0.667	0.652	0.500	0.550	0.514	0.590
LogDet-MSC	**0.955**	**0.971**	**0.938**	**0.944**	**0.942**	**0.947**

4 Educational Activities

New course development. The PI is planning to establish a graduate topic course: "Multi-dimensional Data Set Representations and Applications" that currently is not available at Southern Illinois University. The goals are to introduce students to new developments in big data analysis and to encourage graduate

students to build up their own research topics/theses, with focusing on the discovery of new mathematical tools for interdisciplinary applications. The course will be offered during the period of this research project by PI for graduate students in College of Science and Engineering at PI's Institute. This is part of efforts for curriculum development by the department of Mathematics.

Table 8: Clustering results(mean) on ORL. We set $\lambda = 0.2$ in proposed *LogDet-MSC*.

Method	NMI	ACC	AR	F-score	Precision	Recall
SPC_{best}	0.884	0.725	0.655	0.664	0.610	0.728
LRR_{best}	0.895	0.773	0.724	0.731	0.701	0.754
RMSC	0.872	0.723	0.645	0.654	0.607	0.709
DiMSC	0.940	0.838	0.802	0.807	0.764	0.856
LTMSC	0.930	0.795	0.750	0.768	0.766	0.837
t-SVD-MSC	0.993	0.970	0.967	0.968	0.946	0.991
LogDet-MSC	**0.995**	**0.976**	**0.976**	**0.977**	**0.963**	**0.991**

Table 9: Clustering results(mean) on Scene-15. We set $\lambda = 0.005$ in proposed *LogDet-MSC*.

Method	NMI	ACC	AR	F-score	Precision	Recall
SPC_{best}	0.421	0.437	0.270	0.321	0.314	0.329
LRR_{best}	0.426	0.445	0.272	0.324	0.316	0.333
RMSC	0.564	0.507	0.394	0.437	0.425	0.450
DiMSC	0.269	0.300	0.117	0.181	0.173	0.190
LTMSC	0.571	0.574	0.424	0.465	0.452	0.479
t-SVD-MSC	0.858	0.812	0.771	0.788	0.743	0.839
LogDet-MSC	**0.980**	**0.991**	**0.981**	**0.982**	**0.984**	**0.981**

Graduate student training. PI currently is supervising two graduate students, one is a Ph.D. candidate from Computer Science, and the other one is a Ph.D. candidate from Mathematics. Both of their research are closely related to this proposal. The student from computer science is working on the tensor low rank approximation, aiming at handling big data set in an efficient way. The student from mathematics is to study the Markowitz Portfolio Theory by incorporating the mean, variance, skewness, and kurtosis of both return and liquidity into an investors objective function, which is formulated as a 4th-order tensor problem. Both students are expected to participate in the proposed project extensively. The brief time frame is set in the following: during the first year, we will develop theoretical foundation under the framework of optimization theory in vector space, focusing on the aforementioned approaches. Formal and extensive numerical experiments will be conducted in the second year. The third year will focus on the improvement of achieved outcomes and connect them to real-world problems in our best effort.

Undergraduate recruiting. PI does not have undergraduate students at this point but will participate in various programs designed to enrich the undergraduate research experience. Institutional support for research at SIUC is provided by grants through the Research Enriched Academic Challenge program and Undergraduate Assistantships. Grant-supported opportunities to enhance research skills and experiences for undergraduates include the NSF Louis Stokes Alliance for Minority Participation, McNair Program, and NSF Scholarships in STEM. SIUC also has an active Learning Living Community where PI can meet with undergraduates and facilitates their involvement in professional development activities. This

contact provides an excellent means of identifying promising students to participate in this program to develop appropriate undergraduate projects.

Mentoring high school students. PI currently mentors three high school students and involves in related research programs. For example, PI had been worked with Irena Gao (Senior, Illinois Mathematics and Science Academy) in a project for the study of kidney exchange algorithm, which entered the regional semifinal of Siemens Competition lats Fall. Two other students (junior, Carbondale Community High School) work on the identification of diffusion rate for insect unrestricted random walks via using boundary population distribution (a data handling problem related to statistics), and will participate in Illinois Science Fair next Spring. PI will commit to public outreach by recruiting undergraduate interns in their research program, including students from Brehm Preparatory School with learning disabilities (some are working for our previous NSF projects). Their roles are mainly to help literature search and to record experiment data from extensive simulations with writing experiment reports. PI also has been served as MATHCOUNTS head coach for a local elementary school (Unity Point Elementary School) since 2014, and this project can provide some introductory exposition of mathematics by simulations and pictures (such as Multi-view problems) for young students, promoting mathematical learning interests in our local community, which is located in a rural area at Southern Illinois with many low-income families.

Intellectual Merit

The objective of this proposal is to develop the non-convex regularization for low rank approximation of multi-dimensional datasets, for which the convex approach appears to be limited. Currently, there are very few studies on non-convex regularization for tensor related problems. To formulate these problems under vector space theory, such as introducing Fourier transform and induced tensor norm, can enrich mathematical theory and tools that lead to better solutions, and low rank approximation of tensors can be addressed by the optimization theory in vector space, leading to expanding our research capacity.

Broader Impacts

Educational Impacts. Nowadays interdisciplinary research is becoming a main theme in scientific community and education should reflect this new status of the development. The study of multi-dimensional dataset problems falls very well into the interdisciplinary area. Thus this research program can generate many (large or small) projects that are suitable for graduate/undergraduate students, even for high school students. Professional development of the students will be enhanced through mentoring activities and development of graduate topic course, to promote the development of mathematical theory and tools.

Applicational Prospective. During the last decade, the study of tensor competition, tensor robust PCA, and multi-view clustering is becoming very demanding in signal processing, image processing, and computer vision, ...etc., and has drawn considerable attention across the scientific community. As the convex method appears to be limited, non-convex approach become necessary in order to meet the challenges from various applications. Our preliminary tests demostrate that the proposed approach is applicable.
Outreaching middle/high school students. We have many very talent K-12 students at Southern Illinois. Unlike urban areas, there are very limited resources available to our local middle/high school students, such as extra math curriculum, for helping their intellectual growth. The proposed project will increase the opportunities for local young students to participate in various activities related to mathematics. The PI has an extensive tracked record for working with local students in learning mathematics.

5 Prior NSF Support DMS-1419028

(a) DMS-1419028 (Co-PI: Dr. J. Xu), \$90, 000, September 1, 2014 –August 31, 2018.
(b) the title of the project: *Numerical Approximation of Joint Spectral Radius by Lower Rank Matrix Sets.*
(c) Summary of the results of the completed work. Due to space limitation, we here only provide those results which are the most relevant to this proposal. The characteristics of tensor decomposition in certain desirable forms is its uniqueness which appears to be essential in many applications. Extensive work under the framework of algebraic geometry has provided many fundamental results associated with tensor rank and dimension to ensure the generic identiability. Different from most current approaches in literature, in [41], by using algebraic geometry theory, we study the rank L_r and rank $L_r \otimes$ rank-1 block term tensor decomposition for higher order tensors. In particular, we establish the sufficient and necessary conditions for a general tensor to have finitely many block term decomposition, and sufficient conditions to guarantee a unique decomposition. In [40], we study a specific big data model via multilinear rank tensor decompositions. The model approximates to a given tensor by the sum of multilinear rank $(1, L_r, L_r)$ terms. And we characterize the identifiability property of this model from a geometric point of view. Our main results consists of exact identifiability and generic identifiability. The arguments of generic identifiability relies on the exact identifiability, which is in particular closely related to the well-known "trisecant lemma" in the context of algebraic geometry. This connection discussed in this paper demonstrates a clear geometric picture of this model. Stochastic matrices play an important role in the study of probability theory and statistics, and are often used in a variety of modeling problems in economics, biology and operation research. Recently, the study of tensors and their applications became an important topic in numerical analysis and optimization. In [31], we focus on studying stochastic tensors and, in particular, we study the extreme points of a set of multi-stochastic tensors. Two necessary and sufficient conditions for a multi-stochastic tensor to be an extreme point are established. These conditions characterize the generators of multi-stochastic tensors. An algorithm to search the convex combination of extreme points for an arbitrary given multi-stochastic tensor is developed. Based on our obtained results, some expression properties for third-order and n-dimensional multi-stochastic tensors (n=3 and 4) are derived, and all extreme points of 3-dimensional and 4-dimensional triply-stochastic tensors can be produced in a simple way. As an application, a new approach for the partially filled square problem under the framework of multi-stochastic tensors is given.
Intellectual Merit: The completed work has addressed the low-rank approximations of joint spectral radius for a finite set of matrices as well as established a sequence of results for tensor related problems and optimal control problems of PDEs, associated with the development of fast solvers.
Broader Impacts: Developed two new graduate courses and finished supervision of four Ph.D. and two master graduate students (both PI and Co-PI), conducted outreaching to local middle/high school students extensively.
(d) Publications resulting from prior award
Thirty (30) journal papers have been published or accepted to appear in addition to seven (7) international conference papers. These publications have been listed in the end of section References. **(e)** N/A.
(f) A description of the relation of the completed work to the proposed work.
Our completed work has not only set up a solid foundation for the current proposed work, but also provided an effective approach for the study of low rank approximation for tensors. In order to study the highly multi-dimensional datasets, higher-order tensor rank approximation has to be approximated by low-order tensor recursively, and eventually relies on matrix rank approximation once the tensor structure is adequately retained. Therefore our previous project closely connects to the current proposed one.

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Publication list resulting from prior award

1. M. YANG, W. LI and M. XIAO, *On identifiability of higher order block term tensor decomposition of rank L_r and rank $L_r \otimes$ rank-1*, Linear and Multilinear Algebra, doi: 10.1080/03081087.2018.1502251, available online, July 31, 2018.

2. N. YAO and M. XIAO, *Asymptotic analysis for affine point processes with large initial intensity*, Analysis and Application, doi:10.1142/S0219530518500197, available online, May 12, 2018.

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14. J. LIU AND M. XIAO, *A Leapfrog semi-smooth Newton-multigrid method for semilinear parabolic optimal control problems*, Computational Optimization and Applications, Volume 63, Issue 1, pp 69-95, 2015.

15. J. LIU, Y. HUANG, H. SUN, AND M. XIAO, *Numerical Methods for Weak Solution of Wave Equation with van der Pol Type Nonlinear Boundary Conditions*, Numerical Methods for Partial Differential Equations, Volume 32, Issue 2, pp. 373-398, 2016.

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30. J. Xu and P. Rajasingam, *New unified matrix upper bound on the solution of the continuous coupled algebraic Riccati equation*, Journal of the Franklin Institute, 353, pp. 1233-1247, 2016

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Biographical Sketches of MingQing Xiao

Professional Preparation

1. Guangdong University of Technology, China, Mathematics, B.S., 1983.
2. Zhongshan University, China, Applied Mathematics, M.S., 1989.
3. University of Illinois at Urbana-Champaign, Computational Science & Engineering, M.S., 1995.
4. University of Illinois at Urbana-Champaign, Ph.D., Applied Mathematics, 1997.
5. University of California at Davis, Postdoc, Nonlinear Control and Estimation, 1997-1999.

Appointments

1. Guest Professor, Shenzheng University, China (since 2016).
2. Guest Professor, University of Electronic Science and Technology of China (since 2015).
3. Associate Editor, European Journal of Control (since 2012).
4. Associate Editor, Auomatica (2008-2011).
5. Guest Professor, Guangdong University of Technology, China (since 2010).
6. Professor, Southern Illinois University at Carbondale (August 2007 - present).
7. Associate Editor, IEEE Trans. Automat. Control (2003-2006).
8. Associate Professor, Southern Illinois University, Carbondale, IL (July 2002-July 2007).
9. Assistant Professor, Department of Mathematics, Southern Illinois University, Carbondale (January 2000-June 2002).
10. Summer Faculty Fellow, US Air Force Research Laboratory, Wright-Patterson, Summer 2002, 2001.
11. Visiting Research Assistant Professor, Department of Mathematics, University of California, Davis. (July 1997-December 1999)
12. Research/teaching Assistant, University of Illinois, Urbana (1992-1997).
13. Visiting Scholar, Board of Studies in Applied Mathematics, Chinese University of Hong Kong, 1990.
14. Research Fellow and Principle Instructor, Zhongshan University, China (1989-1991).

Publications Most Relevant to the Proposed Project

1. M. YANG, W. LI and M. XIAO, *On identifiability of higher order block term tensor decomposition of rank L_r and rank $L_r \otimes$ rank-1*, Linear and Multilinear Algebra, doi: 10.1080/03081087.2018.1502251, available online, July 31, 2018.
2. M. YANG, D. CHE, W. LIU, Z. KANG, C. PENG, M. XIAO, and Q. CHENG, *On identifiability of 3-tensors of multilinear rank* $(1, L_r, L_r)$, Big Data and Information Analytics, American Institute of Mathematical Science, 1(4): 391-401, 2016.
3. R. KE, W. LI, and M. XIAO, *Characterization of extreme points of multi-stochastic tensors*, Computational Methods in Applied Mathematics, 16(3):459-474, 2016.
4. B. LI, J. LIU, and M. XIAO. *A fast and stable preconditioned iterative method for optimal control problem of wave equations*. SIAM J. on Scientific Computing, 37(6), pp. A2508-A2534, 2015.

5. J. LIU and M. XIAO, *Rank-one characterization of joint spectral radius of finite matrix family*, Linear Algebra and its Applications, 438(8):32583277, 2011.

Other Significant Publications

1. J. LIU and M. XIAO. *A new semi-smooth Newton multigrid method for control-constrained semi-linear elliptic PDE problems*, Journal of Global Optimization, 64(3), pp. 451-468, 2016.

2. L. LI, Y. HUANG, and M . XIAO. *Observer design for wave equations with van der Pol type boundary conditions*, SIAM J. Control Optim., 50(3), pp. 1200-1219, 2012.

3. M. XIAO. *Quantitative characteristic of rotating stall and surge for Moore-Greitzer PDE model of an axial compressor*, SIAM J. on Applied Dynamical Systems, 7(1), pp. 39-62, 2008.

4. A. J. KRENER and M. XIAO. *Nonlinear observer design in the Siegel domain*, SIAM J. Control Optim., 41(3), pp. 932-953, 2002.

5. M. XIAO and T. BASAR. *Finite-dimensional compensators for the H^{∞}-optimal control of infinite-dimensional systems via a Galerkin-type approximation*, SIAM J. Control Optim., 37(5), pp. 1614-1647, 1999.

Synergistic Activities

1. Keynote Speaker: *Interdisciplinary Computing and Optimization (SICO-2017)*, July 21-25, 2017, Ballarat, Australia. Conference is supported in part by the international Society of Global Optimization (iSoGO), and in part by US Air Force Office of Scientific Research (AFOSR).

2. (i) Program Committee, *SIAM Conference on Control and Its Applications*, San Diego, CA,July 8-10, 2013; Paris, France, July 8-10, 2015; Pittsburgh, PA, July 10-12, 2017. (ii) Organizing Committee, *International Federation of Automatic Control* (IFAC) *Nonlinear Control Symposium*, Monterey, CA, August 23-25, 2016.

3. Received *Excellent Scholar of College of Science* award, 2015/2016. Development of curricular: graduate topic course *Matrix Computations* (with Dr. J. Xu, 2014); graduate course *Optimization* (2015); undergraduate course *Mathematical Modeling in Biological Disease* (with Dr. D. Xu, 2014-2016). Served for university Faculty Senate (2012-2015). Two Ph.D. students graduated (2014, 2015) under my supervision.

4. Organizing math enhancement classes for local K-12 students and teaching the classes every Saturday as a volunteer since 2013. Broad member of local Math/Chinese Learning Club for Kids. Judge of Illinois Science Fair for middle/high school students (since 2015).

5. Received first place coach award 2015, 2017, and 2018 respectively, second place coach award 2016, *MATHCOUNTS*, Egyptian Chapter (Southern Illinois area). MATHCOUNTS head coach, *Unity Point Elementary School*, (2014-present).

SUMMARY PROPOSAL BUDGET

YEAR 1

ORGANIZATION **Southern Illinois University at Carbondale**				FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **MingQing Xiao**				AWARD NO.		
A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)	
1. **MingQing Xiao - Professor**	0.00	0.00	1.00	**10,632**		
2.						
3.						
4.						
5.						
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**		
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	**10,632**		
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**		
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**		
3. (**2**) GRADUATE STUDENTS				**3,593**		
4. (**0**) UNDERGRADUATE STUDENTS				**0**		
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**		
6. (**0**) OTHER				**0**		
TOTAL SALARIES AND WAGES (A + B)				**14,225**		
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**6,446**		
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**20,671**		
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				**0**		
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**1,800**		
2. INTERNATIONAL				**0**		
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ **0** 2. TRAVEL **0** 3. SUBSISTENCE **0** 4. OTHER **0**						
TOTAL NUMBER OF PARTICIPANTS (**0**) TOTAL PARTICIPANT COSTS				**0**		
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				**200**		
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**800**		
3. CONSULTANT SERVICES				**0**		
4. COMPUTER SERVICES				**0**		
5. SUBAWARDS				**0**		
6. OTHER				**0**		
TOTAL OTHER DIRECT COSTS				**1,000**		
H. TOTAL DIRECT COSTS (A THROUGH G)				**23,471**		
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 47.5000, Base: 23471)**						
TOTAL INDIRECT COSTS (F&A)				**11,149**		
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**34,620**		
K. SMALL BUSINESS FEE				**0**		
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				**34,620**		
M. COST SHARING PROPOSED LEVEL $ **0**	AGREED LEVEL IF DIFFERENT $					
PI/PD NAME **MingQing Xiao**	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION					
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG			

1 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

YEAR 2

ORGANIZATION **Southern Illinois University at Carbondale**	FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **MingQing Xiao**	AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. **MingQing Xiao - Professor**	0.00	0.00	1.00	10,951	
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	10,951	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (2) GRADUATE STUDENTS				16,571	
4. (2) UNDERGRADUATE STUDENTS				1,600	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				29,122	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				7,044	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				36,166	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				1,800	
2. INTERNATIONAL				0	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				200	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				800	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				1,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				38,966	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 47.5000, Base: 38966)**					
TOTAL INDIRECT COSTS (F&A)				18,509	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				57,475	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				57,475	
M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $					

PI/PD NAME **MingQing Xiao**	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

YEAR 3

ORGANIZATION **Southern Illinois University at Carbondale**	FOR NSF USE ONLY PROPOSAL NO.	DURATION (months) Proposed	Granted	
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **MingQing Xiao**	AWARD NO.			

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. **MingQing Xiao - Professor**	0.00	0.00	1.00	**11,279**	
2.					
3.					
4.					
5.					
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.00	**11,279**	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	
3. (**2**) GRADUATE STUDENTS				**17,068**	
4. (**2**) UNDERGRADUATE STUDENTS				**1,600**	
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	
6. (**0**) OTHER				**0**	
TOTAL SALARIES AND WAGES (A + B)				**29,947**	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**7,256**	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**37,203**	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				**0**	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**1,800**	
2. INTERNATIONAL				**0**	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ **0** 2. TRAVEL **0** 3. SUBSISTENCE **0** 4. OTHER **0**					
TOTAL NUMBER OF PARTICIPANTS (**0**) TOTAL PARTICIPANT COSTS				**0**	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				**200**	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**800**	
3. CONSULTANT SERVICES				**0**	
4. COMPUTER SERVICES				**0**	
5. SUBAWARDS				**0**	
6. OTHER				**0**	
TOTAL OTHER DIRECT COSTS				**1,000**	
H. TOTAL DIRECT COSTS (A THROUGH G)				**40,003**	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 47.5000, Base: 40003)**					
TOTAL INDIRECT COSTS (F&A)				**19,001**	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**59,004**	
K. SMALL BUSINESS FEE				**0**	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				**59,004**	
M. COST SHARING PROPOSED LEVEL $ **0** AGREED LEVEL IF DIFFERENT $					

PI/PD NAME **MingQing Xiao**	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

Cumulative

	FOR NSF USE ONLY		
ORGANIZATION **Southern Illinois University at Carbondale**	PROPOSAL NO.	DURATION (months) Proposed	Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR **MingQing Xiao**	AWARD NO.		

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. MingQing Xiao - Professor	0.00	0.00	3.00	32,862	
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	3.00	32,862	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (6) GRADUATE STUDENTS				37,232	
4. (4) UNDERGRADUATE STUDENTS				3,200	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				73,294	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				20,746	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				94,040	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)					
TOTAL EQUIPMENT				0	
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				5,400	
2. INTERNATIONAL				0	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 0					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				0	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				600	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				2,400	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				3,000	
H. TOTAL DIRECT COSTS (A THROUGH G)				102,440	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				48,659	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				151,099	
K. SMALL BUSINESS FEE				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				151,099	
M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $					

	FOR NSF USE ONLY		
PI/PD NAME **MingQing Xiao**	INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

Budget Justification

MingQing Xiao

Personnel:

PI Salary:

PI will spend a significant portion of his time on this project–including theoretical approaches, numerical algorithm development, data analyzes, and writing of manuscripts. PI is requesting 1 month of summer salary per year. An estimated 3% annual increase in base pay is included in the budget.

Graduate Student Support:

A portion of the proposed research will serve as the basis for two Ph.D. students for their thesis research and related activities. PI requests two graduate students' support for 2 months at 25% effort (.5 person months each) for the first year, and 9 months at 25% effort (2.25 person months each) for the following two years.

Undergraduate Student Support:

This project encourages undergraduate students to participate in the proposed research activities, this includes help in collecting research references, implementing some numerical simulations, and conducting individual research projects. Two undergraduate student support for eight weeks (10 hrs/per wk/per person) for the second and the third year is requested.

Fringe Benefits:

Fringe benefits are included for the PI at the federally negotiated rate of 58.3% for faculty. Primary care fees are included for the graduate students as required by the graduate student union agreement.

Travel:

PI requests funds for attending national conferences (such as SIAM Conference on Computational Science & Engineering, SIAM Conference on Imaging Science, SIAM Conference on Linear and Multi-linear Algebra) that are related to the project once per year. The purpose is to present results and to exchange ideas with colleagues in related areas. The estimated cost per meeting for PI and graduate students is about $1800 (which includes coach-class airfare, hotel accommodations, registration fees, and per diem).

Other Direct Costs:

Publication Costs:

A minimum $800 is requested for each year for publication and duplication costs associated with the project.

Materials and Supplies:

A minimum $200 is requested for each year for small materials that are related to the proposed research.

Indirect Costs:

Indirect costs are calculated at the federally negotiated rate of 47.5% MTDC for on-campus research.

Current and Pending Support

(See PAPPG Section II.C.2.h for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: MingQing Xiao

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title: Study of Low Rank Approximation of Tensorial Data Set via Non-convex Regularization

Source of Support: DMS-NSF
Total Award Amount: $ 151,099 Total Award Period Covered: 05/15/19 - 05/14/22
Location of Project: Southern Illinois University Carbondale
Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 1.00

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support
Project/Proposal Title:

Source of Support:
Total Award Amount: $ Total Award Period Covered:
Location of Project:
Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

 USE ADDITIONAL SHEETS AS NECESSARY

Facilities, Equipment, and Other Resources

MingQing Xiao

The main resource for conducting this proposed research project is the requirement of necessary computing facility. PI's institute, Southern Illinois University Carbondale, has the required computing facilities and equipment for PI to complete this project. There are mainly four level computing resource supports:

Personal Level: PI has two desktops and three laptops that can be used for basic/preliminary computations for this research project at PI's office.

Department Level: The department of mathematics has a large computer lab that is suitable for graduate and undergraduate students to pursue independent research and elementary computing.

College of Science Level: The College of Science has a High Performance Distributed Computing Lab, resided in the Department of Computer Science. It has 10 Dell Linux workstations, 2 iMac systems, and high-end NVidia GPUs for parallel scientific computing. It is suitable for senior graduate students to do some extensive computing.

University Level: IT Research Computing is a team in Southern Illinois University Information Technology who supports computationally intense research activities for the University community. The primary resource is the BigDog high performance computing cluster, a 34.7 TeraFLOPs supercomputer. The team was formed in July, 2013, and they are located in the Northwest Annex, Wing B, Third Floor. The team was formed to centrally assist University researchers with computationally intense projects related to grants and publications and to assist with student research under the supervision of faculty.

Data Management Plan

MingQing Xiao

PI will follow the NSF policy on the dissemination and sharing of research results if this proposal is awarded. More specifically, the proposed project will conform:

(1) all types of data, samples, physical collections, software, curriculum materials, and other materials to be produced in the course of the project will follow the existing policies and standards, and make available to public for research and/or teaching purposes;

(2) PI will follow the policies for access and sharing including provisions for appropriate protection of privacy, confidentiality, security, intellectual property, or other rights or existing requirements;

(3) PI will acknowledge NSF support in any publication (including Web pages) in the NSF specified terms required in AAG Chapter VI.D.4;

(4) PI will also orally acknowledge during all possible news media interview, including popular media such as radio, television and news magazines;

(5) PI will be responsible for assuring that every publication of material (including World Wide Web pages) based on or developed under this award, except scientific articles or papers appearing in scientific, technical or professional journals, contains the NSF required disclaimer shown in AAG Chapter VI.D.4.;

(6) PI will be responsible for assuring that the cognizant NSF Program Officer is provided access to, either electronically or in paper form, a copy of every publication of material based on or developed under this award, clearly labeled with the award number and other appropriate identifying information, promptly after publication;

(7) PI is aware of their obligations in regard to copyrights illustrated in AAG Chapter VI.D.2) and their responsibilities as members of the scientific and engineering community to disseminate and share research results stated in AAG Chapter VI.D.4.